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სახელმწიფო უნივერსიტეტი

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საორგანიზაციო კომიტეტი:

თავმჯდომარე: როლანდ დუდუჩავა, **თავმჯდომარის მოადგილე:** თინათინ დავითაშვილი, **სწავლული მდივანი:** მედეა ცაავა, **წევრები:** თენგიზ ბუჩუკური, გურამ გოგიშვილი, ნუგზარ კერესელიძე, ლევან სიგუა, გიორგი ტეფნაძე, **ადგილობრივი საორგანიზაციო კომიტეტი:** ვლადიმერ ბალაძე, ანზორ ბერიძე, რუსლან ცინარიძე.

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Editors: Guram Gogishvili, Maia Kvinikadze, Maia Japoshvili

Cover Design: David Sulakvelidze

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რევაზ მიხეილის ძე აბსავა დაიბადა 1947 წლის 9 თებერვალს ზუგდიდის რაიონის სოფელ გრიგოლიშში, მოსამსახურის ოჯახში. მამა - მიხეილ აბსავა - განათლებით ისტორიკოსი, პედაგოგი იყო, დედა - ნინა ლოგუა - დიასახლისი.

რევაზ აბსავამ 1965 წელს დაამთავრა კორცხელის საშუალო სკოლა ოქროს მედალზე. 1969 წელს კი ა. მ. გორკის სახელობის სოხუმის სახელმწიფო პედაგოგიური ინსტიტუტის ფიზიკა-მათემატიკის ფაკულტეტი მათემატიკის სპეციალობით.

1971 წლიდან მუშაობდა სოხუმის პედაგოგიური ინსტიტუტის, 1979 წლიდან აფხაზეთის სახელმწიფო უნივერსიტეტის ფიზიკა-მათემატიკის ფაკულტეტის ალგებრა-გეომეტრიის კათედრაზე მასწავლებლის, უფროსი მასწავლებლის, დოცენტის თანამდებობებზე. 1979-1983 წლებში იყო ფიზიკა-მათემატიკის ფაკულტეტის დეკანის მოადგილე.

1989-2004 წლებში (გარდაცვალებამდე) იყო თსუ სოხუმის ფილიალის მათემატიკისა და კომპიუტერულ მეცნიერებათა ფაკულტეტის დეკანი.

რევაზ აბსავა, როგორც მეცნიერი, ივ. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის ალბათობის თეორიისა და მათემატიკური სტატისტიკის კათედრის აღზრდილია. პროფესორ ელიზბარ ნადარაიას ხელმძღვანელობით მან შეასრულა საკანდიდატო დისერტაცია თემაზე: „მრავალგანზომილებიანი განაწილების სიმკვრივისა და რეგრესიის ფუნქციის არაპარამეტრული გულოვანი ტიპის შეფასებათა შესახებ“ (01.01.05 – ალბათობის თეორია და მათემატიკური სტატისტიკა), რომელიც 1986 წლის 10 ოქტომბერს ბრწყინვალედ დაიცვა მოსკოვის ელექტრონული მანქანათმშენებლობის ინსტიტუტის სპეციალიზირებულ საბჭოზე (К.063.68.05) და მიენიჭა ფიზიკა-მათემატიკის მეცნიერებათა კანდიდატის ხარისხი.

1993 წლიდან ბატონი რევაზი თბილისში აგრძელებს თავის სამეცნიერო, პედაგოგიურ და ადმინისტრაციულ საქმიანობას. ამ მძიმე წლებში ის სათავეში ედგა მათემატიკისა და კომპიუტერულ მეცნიერებათა ფაკულტეტს, ალგებრა-გეომეტრიისა და მათემატიკური სტატისტიკის კათედრას.

ბატონი რევაზის სამეცნიერო კვლევის ობიექტს წარმოადგენდა: განაწილების კანონის ფუნქციონალური მახასიათებლების ზოგადი სახის არაპარამეტრულ შეფასებათა კვადრატული გადახრის ზღვარითი განაწილების მიღების მეთოდის შემუშავება და რეალიზება; განაწილების სიმკვრივის და რეგრესიის მრუდის შესახებ მარტივი და რთული ჰიპოთეზების შესამოწმებელი კრიტერიუმების აგება და მათი სიმძლავრის დადგენა; კლასიფიკაციის და იდენტიფიკაციის პრობლემებთან დაკავშირებული ზოგიერთი საკითხის გადაწყვეტა.

ამ პრობლემატიკაში ფუნდამენტური შედეგები მიღებული აქვთ ა. კოლმოგოროვს, ვ. გლივენკოს, ნ. სმირნოვს, მ. როზენბლატს, ე. პარზენს, ნ. ჩენცოვს, ე. ნადარაიას, რ. ჰოლს, ი. იბრაგიმოვს, რ. ხასმინსკის და სხვებს.

კარგად არის ცნობილი, რომ მართვის ამოცანებში განსაკუთრებული ადგილი უჭირავს იდენტიფიკაციას, რაც წარმოუდგენელია მართვის ობიექტის შესაბამისი მოდელის აგების გარეშე. სტატიკური და დინამიური სისტემების იდენტიფიკაციის შინაარსობრივად განსხვავებული პრობლემების ფორმალიზება შესაძლებელია როგორც ფუნქციის აღდგენის ამოცანა, რომლისთვისაც მოცემულია მისი ექსპერიმენტული მნიშვნელობები. ამოცანა განსაკუთრებით რთულდება მაშინ, როცა დამახინჯებულია ექსპერიმენტული მონაცემები. იდენტიფიკაციის პარამეტრული მეთოდები გამოიყენება იმ შემთხვევაში, როცა აღსადგენი დამოკიდებულების მოდელი მოცემულია რამდენიმე პარამეტრით. არაპარამეტრული მეთოდები ასეთ მოდელს არ იყენებენ. ისინი გარკვეულწილად აგლუვებენ ექსპერიმენტულ მონაცემებს, ამასთან, ფუნქცია-შეფასება, საზოგადოდ, არაპარამეტრულია.

ასეთი სახის ტრადიციულ არაპარამეტრულ შეფასებათა რიგს მიეკუთვნება რეგრესიის ნადარაია-ვატსონის და გასერ-მიულერის შეფასებები და განაწილების სიმკვრივის პარზენისა და ჩენცოვის შეფასებები. ზემოთ აღნიშნული ავტორების ნაშრომებში გამოკვლეულია კონკრეტული ამოცანები, დაკავშირებული განაწილების სიმკვრივის და რეგრესიის მრუდის შეფასებების პრობლემატიკასთან და, აგრეთვე, ამ შეფასებების გამოყენებებთან კლასიფიკაციისა და იდენტიფიკაციის ამოცანებში.

ბუნებრივად ჩნდება ზოგადი თეორიის შემუშავების აუცილებლობა, რაც დაკვირვებათა განაწილების კანონის სხვადასხვა ფუნქციონალური მახასიათებლის შეფასებებთან დაკავშირებული პრობლემების ფართო წრის შესწავლის ერთიანი პოზიციით განხილვის შესაძლებლობას იძლევა. სწორედ ეს თეორიაა განვითარებული რევაზ აბსავას სადოქტორო დისერტაციაში „განაწილების კანონის ფუნქციონალური მახასიათებლების არაპარამეტრული შეფასებები და გამოყენება კლასიფიკაციის და იდენტიფიკაციის ამოცანებში“.

მან მარტინგალურ მეთოდებზე დაყრდნობით შეიმუშავა არაპარამეტრული სტატისტიკის გულოვანი შეფასებების და ჰიპოთეზათა შემოწმების ეფექტური მეთოდები. ამან მისცა საშუალება ერთიანი მიდგომით გადაეწყვიტა მრავალი პრობლემა და, რაც მთავარია, კლასიფიკაციისა და იდენტიფიკაციის თეორიის საინტერესო ამოცანები.

რევაზ აბსავა 30-ზე მეტი სამეცნიერო ნაშრომის ავტორია, რომელთა შორის არის მონოგრაფია, სახელმძღვანელო ალბათობის თეორიაში და ორი მეთოდური მითითება. მისი ნაშრომები გამოქვეყნებულია უცხოეთის და საქართველოს ავტორიტეტულ სამეცნიერო ჟურნალებში. ის იყო მრავალი საერთაშორისო და ადგილობრივი კონფერენციის მონაწილე.

რეზოს შრომები არის ციტირებადი და მათი ციტირება დღითიდღე იზრდება. ეს იმის დასტურია, რომ რევაზ აბსავას სახელი ჩაიწერა მათემატიკური სტატისტიკის ისტორიაში.

რევაზ აბსავას მეცნიერულმა კვლევებმა დასრულებული სახე მიიღო სადოქტორო დისერტაციის სახით: „განაწილების კანონის ფუნქციონალური მახასიათებლების არაპარამეტრული შეფასებები და გამოყენება კლასიფიკაციის და იდენტიფიკაციის ამოცანებში“ (01.01.08 – მათემატიკური კიბერნეტიკა). 2004 წლის 28 მარტს ფიზიკა-მათემატიკის მეცნიერებათა დოქტორის სამეცნიერო ხარისხის მოსაპოვებლად დისერტაცია წარედგი-

ნა ივ. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის სადისერტაციო საბჭოს.

28 მაისს გამოყვეს ოფიციალური ოპონენტები: პროფესორი ალბერტ შირიაევი, რუსეთის მეცნიერებათა აკადემიის წევრ-კორესპონდენტი (მოგვიანებით, 2011 წლიდან რუსეთის მეცნიერებათა აკადემიის აკადემიკოსი), ვ. ა. სტეკლოვის სახელობის მათემატიკის ინსტიტუტის შემთხვევითი პროცესების სტატისტიკის ლაბორატორიის გამგე; პროფესორი ომარ ღლონტი და პროფესორი ბესარიონ დოჭვირი.

დაინიშნა დაცვის დღე - 22 ოქტომბერი... მაგრამ ვერ მოესწრო ამ დღეს, რევაზი უეცრად გარდაიცვალა 57 წლის ასაკში, 2004 წლის 29 მაისს.

მომხდართან დაკავშირებით რუსეთის მეცნიერებათა აკადემიის აკადემიკოსი ალბერტ შირიაევი წერდა: „...ცოტა ხნის წინ გავეცანი რეზოს სადოქტორო დისერტაციის მასალებს და დავწერე რეცენზია, საიდანაც ნათელია, თუ როგორი შესანიშნავი ნაშრომი მოამზადა მან. ის აქეთკენ მოდიოდა მრავალი წლის განმავლობაში, ღებულობდა რა სულ ახალ და ახალ შედეგებს. რეზომ დაისახა მიზანი და მიაღწია მას, მაგრამ აღმოჩნდა, რომ იმდენი ძალა იყო გახარჯული, უკანასკნელი ნახტომისთვის არ შემორჩა. ჩვენ ყველანი როგორ ველოდით ამ დამსახურებულ მომენტს, მომენტს, რომელიც დაიმსახურა რეზომ თავისი შრომით, თავისი მეცნიერული შედეგებით, მისი პიროვნების არაჩვეულებრივობითა და მიმზიდველობით...

ბევრი დაკარგა მეცნიერულმა საზოგადოებამ, დაკარგა შესანიშნავი ორგანიზატორი... ძალიან, ძალიან სამწუხაროა, როდესაც მიდიან ისეთი ადამიანები, როგორიც რეზოა.“

მოხდა უპრეცედენტო შემთხვევა, დანიშნულ დროს, 2004 წლის 22 ოქტომბერს, თსუ ი. ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტში, თბილისის სახელმწიფო უნივერსიტეტის სადისერტაციო საბჭოს სხდომაზე შედგა რევაზ აბსავას სადოქტორო დისერტაციის დაცვა. დისერტანტის როლში იყო სამეცნიერო კონსულტანტი, საქართველოს მეცნიერებათა აკადემიის წევრ-კორესპონდენტი, პროფესორი ელიზბარ ნადარაია. სადისერტაციო საბჭომ გარდაცვალების შემდეგ ფიზიკა-მათემატიკის მეცნიერებათა დოქტორის სამეცნიერო ხარისხი მიანიჭა ქართველ მათემატიკოსს, რევაზ აბსავას.

ელიზბარ ნადარაიას ძალისხმევით გამოიცა მონოგრაფია: „დაკვირვებათა განაწილების კანონის ფუნქციონალური მახასიათებლების არაპარამეტრულ შეფასებათა თეორიის ზოგიერთი ამოცანა“ (თბილისი, გამომცემლობა თსუ, 2005 და 2008 წლები).

მისი კოლეგები აღნიშნავენ, რომ ბატონი რეზო იყო: კეთილ საქმეებში - შემოუსაზღვრელი; შეცდომებში - უსასრულოდ მცირე; ხასიათით მდგრადი; თანამშრომლების მიმართ საკმაოდ გლუვი; ფაკულტეტისათვის - გამაერთიანებელ-მაინტეგრებელი. ასევე უბრალო, სათნო, ქცევით თავმდაბალი, ხოლო ზნეობრივად მაღალი... საოცარი ღიმილი ჰქონდა... რეზო აბსავა მეცნიერებაში მეცნიერებათა დოქტორი იყო, უმაღლესი განათლების სფეროში - პროფესორი, ხოლო მეგობრობაში - აკადემიკოსი!

კოლეგები, მეგობრები

Professor Revaz Absava – 70

Revaz Mikheil Absava was born on February 9, 1947 in village Grigolishi, Zugdidi Municipality. His father, Mikheil Absava was a teacher of History, mother – a housewife.

In 1965 Revaz Absava successfully finished the secondary school of Kortskeli and was awarded a gold medal. In 1969 he graduated the Faculty of Physics and Mathematics at A. M. Gorky Sokhumi State Pedagogical University having specialized in Mathematics.

From 1971 Revaz Absava worked at Sokhumi Pedagogical Institute. Since 1979 he worked as a lecturer, a senior lecturer and a docent at the Chair of Algebra-Geometry of the Faculty of Physics and Mathematics at State University of Abkhazia. In 1979–1983 he was a Deputy Dean of the Faculty of Physics and Mathematics.

In 1989–2004 (ante mortem) Revaz Absava held the position of Dean of the Faculty of Mathematics and Computer Sciences at Sokhumi Branch of Tbilisi State University.

Revaz Absava, as a scientist, had finished the school of the Chair of Probability Theory and Mathematical Statistics of Iv. Javakhishvili Tbilisi State University. Under guidance of Professor Elizbar Nadaraia he wrote thesis "On the Nonparametric Kernel Estimation of Density and Regression Function of Multidimensional Distribution" (01.01.05 – Probability Theory and Mathematical Statistics) for his Candidate Dissertation, which he successfully defended on October 10, 1986 before the Specialized Council (K.063.68.05) of Moscow Institute of Electronic Machine Building and was awarded the degree of Candidate of Sciences of Physics and Mathematics.

After 1993 R. Absava continued his scientific, pedagogical and administrative works in Tbilisi. During those hard times he headed the Faculty of Mathematics and Computer Sciences, the Chair of Algebra-Geometry and Mathematical Statistics.

The subjects of his studies were: elaboration and application of the method for the marginal distribution of quadratic deviation of general nonparametric estimation of functional properties of the distribution law; construction and determination of the stringency of the criteria for testing simple and complex hypotheses about distribution density and regression curve; solution of some issues of classification and identification problems.

For solution of these problems there are fundamental results obtained by A. Kolmogorov, V. Glivenko, N. Smirnov, M. Rozenblat, E. Parzen, N. Chentsov, E. Nadaraia, R. Holl, I. Ibragimov, R. Khasmisnky and etc.

It is well known that identification plays an essential role in the control problems. Identification is impossible without construction of relevant control object. Formation of essentially different problems of identification of statistical and dynamic systems is possible in the form of function recovery problem, for which its experimental values are given. A problem becomes especially difficult when its experimental values are distorted. The parametric methods of identification are used when several parameters of recovery correlation model are presented. Such models are not used in nonparametric methods. They, to some extent, equalize experimental data. Besides, function-assessment is generally nonparametric.

The Nadaraia–Watson Regression Estimation, Gasser–Mueller Regression Estimation and Distribution Density Estimation by Parzen and Chentsov belong to above mentioned traditional nonparametric estimations. The works of these authors consider concrete problems of distribution density and regression curve estimations and also application of these estimations in the problems of classification and identification.

Naturally, there is a necessity for elaboration of a general theory, which enables full consideration of studies of wide range problems in the estimations of different functional properties of the observation distribution law. This theory is developed in Revaz Absava's Doctorate Dissertation.

He elaborated effective methods for kernel estimation of nonparametric statistics and hypotheses test on the basis of marginal methods. This enabled him to solve many problems, especially, some interesting problems of classification and identification by integrated approach.

Revaz Absava is an author of more than 30 scientific works, among them a monograph, a textbook on Probability Theory and two methodical instructions. His works have been published in foreign and Georgian highly-ranked scientific journals. He participated in numerous international and local conferences.

R. Absava's works are quoted and their quotations are increasing day after day. It means that Revaz Absava's name has been recorded in the History of Mathematical Statistics.

Revaz Absava's scientific researches were accomplished in the form of Doctorate Dissertation: "Nonparametric Estimations and Application of the Functional Properties of the Distribution Law in the Classification and Identification Problems". On March 28, 2004 the dissertation was introduced to the Dissertation Council of Iv. Javakhishvili Tbilisi State University.

On May 28 official opponents were appointed: Professor Albert Shiryaev, a Corresponding Member of the Russian Academy of Sciences (later, since 2011 an Academician of the Russian Academy of Sciences), the Head of Laboratory of the Statistics of Random Processes at V. A. Steklov Institute of Mathematics, Professor Omar Ghlonti and Professor Besarion Dochviri.

The Dissertation Defense date was appointed – 22 October . . . However, Revaz could not live till that date. He untimely passed away at the age of 57, on May 29, 2004.

Albert Shiryaev, Academician of the Russian Academy of Sciences wrote: "Recently I read Revaz Absava's Dissertation material and wrote a review, which makes it obvious that this work is just significant. R. Absava strived to this goal during many years and obtained new results every time. He had established a goal, but had taken such a hard effort that had left no force to come to the end. We, all were looking forward this great moment, which Revaz Absava earned with his hard work, his scientific results, his extraordinary and charismatic personality…

The scientific society has lost too much as far as he was a great organizer.

It is very pity that such people as Revaz Absava, leave us."

An unprecedented case took place. On the appointed date, October 22, 2004 at the Session of the Dissertation Council Ph. M. 01 08, № 8 of Iv. Javakhishvili Tbilisi State University in TSU I. Vekua Institute of Applied Mathematics Revaz Absava's Doctoral Dissertation was defended. The dissertation was defended by Elizbar Nadaraia, Scientific Consultant, Corresponding Member of Georgian Academy of Sciences. The Dissertation Council awarded the degree of a Doctor of Sciences 01.01.08 of Physics and Mathematics to Georgian Mathematician Revaz Absava after his death.

Monograph "Some Problems of Nonparametric Estimation Theory of the Functional Properties of Observation Distribution Law" was published through Elizbar Nadaraia's effort (Tbilisi, TSU, 2005, 2008).

Revaz Absava's colleagues note that his noble deeds were limitless, his mistakes were extremely little, his behavior was always exemplary, his attitude was favourable towards his coworkers; he was an integrator at the faculty and finally, he was just a merciful, pleasant, modest and a man of high morale… He always had a bright smile on his face… He was a Doctor of Sciences in science, A Professor in the sphere of highest education, and an Academician – in friendship.

Colleagues and friends

Abstracts of Plenary and Invited Speakers

პლენარული და მოწვეული მომხსენებლების თეზისები

On Algebraic and Geometric Properties of Continuous Maps and Applications

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The covariant and contravariant functors having some main properties of classical homology and cohomology functors [8] are playing essential role in the investigation of various topics of homology theory (see [2]).

The purpose of this report is to discuss the properties of continuous maps from the standpoint of geometric topology and algebraic topology. Using a direct system approach and an inverse system approach of continuous maps ([1]–[3], [5], [7]) V. Baladze studied the (co)shape and (co)homological properties of continuous maps. Applications of the here obtained results include the constructions of fiber shape theory of maps of compact Hausdorff spaces and axiomatic approach to Čech spectral and Chogoshvili projective homology theories ([4], [6]) in the sense of Hu [6] for the category of compact Hausdorff spaces, without using the relative groups. Some results connected with this axiomatic characterizations of Čech and Chogoshvili (co)homology theories by system of axioms of Hu where announced in [2]. The problem of characterizations of spectral and projective homology theories by Hu system axioms was formulated by V. Baladze [2].

The coauthors would like to thank V. Baladze for her advise, support and stimulating discussion.

MSC 2000: 54C15, 54C20, 54C56.

Keywords: Approximation of maps, Shape, Coshape, Long exact homology and cohomology sequences of maps, Axioms of homology and cohomology theories in the sense of S.-T. Hu.

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Mixed and Crack Type Dynamical Problems of the Thermopiezoelectricity Theory Without Energy Dissipation

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This paper, we study mixed and crack type boundary value dynamical problems of the linear theory of thermopiezoelectricity for bodies with inner structure. The model under consideration is based on the Green-Naghdi theory of thermo-piezoelectricity without energy dissipation. This theory permits propagation of thermal waves at finite speed. We investigate a mixed boundary value problem for homogeneous isotropic solids with interior cracks. We derive Green's formulae and prove the corresponding uniqueness theorem. Using the Laplace transform, potential method and theory of pseudodifferential equations on a manifolds with boundary we prove existence of solutions and analyze their asymptotic properties. We describe the explicit algorithm for finding the singularity exponents of the thermo-mechanical and electric fields near the crack edges and near the curves, where different types of boundary conditions collide.

This is joint work with T. Buchukuri and D. Natroshvili.

Mixed Boundary Value Problems for the Laplace–Beltrami Equation

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Let $\mathcal{C}$ be a smooth hypersurface in $\mathbb{R}^3$ with a smooth boundary decomposed into two connected $\partial\mathcal{C} = \Gamma = \Gamma_D \cup \Gamma_N$ and non-intersecting $\Gamma_D \cap \Gamma_N = \varnothing$ parts. Let $\nu(\omega) = (\nu_1(\omega), \nu_2(\omega), \nu_3(\omega))^\top$, $\omega \in \overline{\mathcal{C}}$, be the unit normal vector field on the surface $\mathcal{C}$ and consider the Laplace–Beltrami operator written in terms of the Günter's tangent derivatives

$$\Delta_{\mathcal{C}} := \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2, \ \ \mathcal{D}_j := \partial_j - \nu_j \partial_\nu, \ \ j = 1, 2, 3,$$
$$\partial_\nu = \sum_{j=1}^{3} \nu_j \partial_j.$$

Let

$$\nu_\Gamma(t) = (\nu_{\Gamma,1}(t), \nu_{\Gamma,2}(t), \nu_{\Gamma,3}(t)), \ \ t \in \Gamma,$$

be the unit normal vector field on the boundary Γ, which is tangential to the surface $\mathcal{C}$ and directed outside of the surface. We study the following mixed boundary value problem for the Laplace–Beltrami equation

$$\begin{cases} \Delta_{\mathcal{C}} u(t) = f(t), \ \ t \in \mathcal{C}, \quad u^+(\tau) = g(\tau), \ \ \tau \in \Gamma_D, \\ (\partial_{\nu_\Gamma} u)^+(\tau) = h(\tau), \ \ \tau \in \Gamma_N, \quad \partial_{\nu_\Gamma} := \displaystyle\sum_{j=1}^{3} \nu_{\Gamma,j} \mathcal{D}_j. \end{cases} \tag{1}$$

Lax–Milgram Lemma, applied to the BVP (1), gives that it has a unique solution in the classical setting $f \in \widetilde{\mathbb{H}}^{-1}(\mathcal{C})$, $g \in \mathbb{H}^{1/2}(\Gamma)$, $h \in \mathbb{H}^{-1/2}(\Gamma)$.

But in some problems, for example in approximation methods, it is important to know the solvability properties in the non-classical setting

$$f \in \widetilde{\mathbb{H}}_p^{s-2}(\mathcal{C}), \ \ g \in \mathbb{W}_p^{s-1/p}(\Gamma), \ \ h \in \mathbb{W}_p^{s-1-1/p}(\Gamma), \ \ 1 < p < \infty, \ \ s > \frac{1}{p}\,. \tag{2}$$

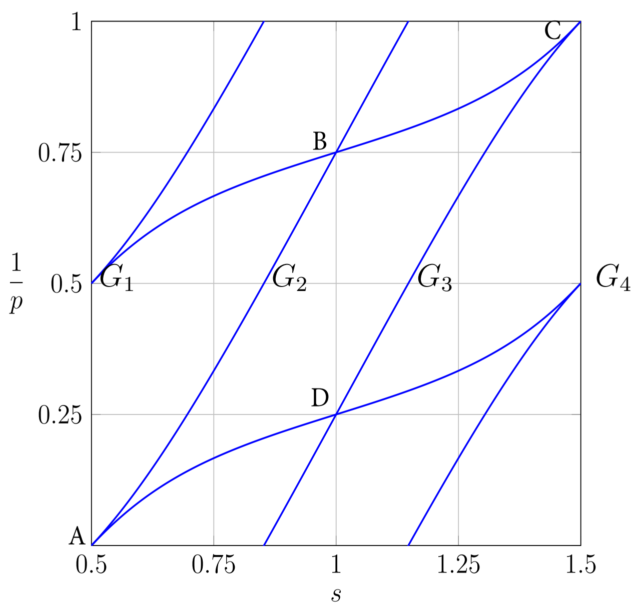

Figure 1. Solutions to the equation (3).

Theorem. *Let $1 < p < \infty$, $s > 1/p$. The BVP* (1) *is Fredholm in the non-classical setting* (*see* (3)) *if and only if*

$$\cos^2 \pi s - \left| \sin 2\pi \left(s - \frac{1}{p} \right) \right| \neq 0. \quad (3)$$

In other words the curves on Figure 1 *does not cross the point $(s-k, 1/p)$, where $k = 0, 1, \ldots$ is an integer such that $\frac{1}{2} < s - k \leqslant \frac{3}{2}$.*

In particular, the BVP (1) *has a unique solution u in the non-classical setting* (2) *if $s - 1/p$ coincided with its fractional part $s - 1/p = \{s - 1/p\}$ and the point $(s - 1/p, 1/p)$ belongs to the open curved quadrangle ABCD on Figure* 1.

The Dynamical Structure of the QCD Ground State

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Using the effective potential approach for composite operators [1], we have formulated a general method of calculation of the truly non-perturbative Yang–Mills vacuum energy density (VED), which is, by definition, the bag constant apart from the sign. It is the main dynamical characteristic of the QCD ground state. Our method allows one to make it free of all the types of the perturbative contributions ('contaminations'), by construction. We also perform an actual numerical calculation of the bag constant as a function of the mass gap. It is this which is responsible for the large-scale dynamical structure of the

QCD ground state [2]. The existence of the stable (being in the stationary state with minimum energy) purely transversal virtual gluon field configurations has been explicitly shown. Using further the trace anomaly relation, we develop a general formalism which makes it possible to relate the bag constant to the gluon condensate defined at the same β function (or, equivalently, effective charge) which has been chosen for the calculation of the bag constant itself. Our numerical result for it shows a good agreement with other phenomenological estimates of the gluon condensate. We have argued that the calculated bag constant may contribute to the dark energy density. Its contribution is by 10 orders of magnitude better than the estimate from the Higgs field's contribution. We also propose to consider the bag energy as a possible amount of energy which can be released from the QCD ground state by a single cycle. The QCD ground state is shown to be an infinite and hence a permanent reservoir of energy.

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Matrix Centralizers and Their Applications

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For a matrix $A \in M_n(\mathbb{F})$ its centralizer

$$\mathcal{C}(A) = \big\{X \in M_n(\mathbb{F})|\ \ AX = XA\big\}$$

is the set of all matrices commuting with A. For a set $S \subseteq M_n(\mathbb{F})$ its centralizer

$$\mathcal{C}(S) = \big\{X \in M_n(\mathbb{F})|\ \ AX = XA \text{ for every } A \in S\big\} = \bigcap_{A \in S} \mathcal{C}(A)$$

is the intersection of centralizers of all its elements. Centralizers are important and useful both in fundamental and applied sciences.

A non-scalar matrix $A \in M_n(\mathbb{F})$ is *minimal* if for every $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \supseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$. A non-scalar matrix $A \in M_n(\mathbb{F})$ is *maximal* if for every non-scalar $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \subseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$.

We investigate and characterize minimal and maximal matrices over arbitrary fields.

Our results are illustrated by applications to the theory of commuting graphs of matrix rings, to the preserver problems, namely to characterize commutativity preserving maps on matrices, and to the centralizers of high orders.

The talk is based on several joint works with G. Dolinar, B. Kuzma, and P. Oblak.

On Caveney–Nicolas–Sondow Hypothesis for Gronwall Numbers

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In 1913 T. Gronwall [1] introduced the numbers $G(N) := \sigma(N)/N \log\log N$, where $\sigma(N)$ and $\log x$ stand for the sum of all divisors of an integer $N > 1$ and natural logarithm of $x > 0$, respectively, and established the limit relationship:

$$\limsup_{N\to\infty} G(N) = e^{\gamma} = 1.78107\ldots, \tag{1}$$

where $\gamma = 0.57721\ldots$ is the Euler–Masceroni constant. S. Ramanujan proved in 1915 (published in 1997) that *if Riemann Hypothesis* **(RH)** *holds true then* $G(N) < e^{\gamma}$ *for all sufficiently large* N. The next step was made by G. Robin [2] who obtained in 1984 much stronger result of principle significance, namely:

$$(\mathbf{RH}) \text{ iff } G(N) < e^{\gamma} \text{ for all integers } N > N_0 := 2^4 \cdot 3^2 \cdot 5^1 \cdot 7^1 = 5040. \tag{2}$$

Let us call an integer $N > 1$ to be *G-improvable*, ($N \in \mathbf{I}$) *iff either:*

($\mathbf{I}_/$): **there is a prime** $q < N$, $q \mid N$, **such that** $G(N/q) > G(N)$,

or:

($\mathbf{I}_\times$): **there is an integer** $A > 1$ **such that** $G(AN) > G(N)$.

Note that the numbers 3, 4 and 5 **are not** *G-improvable.*

Theorem. *Every integer* $N > 5$ *is G-improvable.*

Ascertaining the equivalence of this proposition with both parts of (2) is the main substance of [3], where excellent exposition and exhausted historical comments may be found. Thus the *Theorem stated above implies the validity of* **(RH)**.

The **key idea** of the Theorem's proof is to consider the special class $\mathbf{U_1}$ of all those $N^* > 5$ which **cannot be** G**-improved** by neither division nor multiplication by **any single prime**. Note that this class is not empty and $\min\{N^* : \; N^* \in \mathbf{U_1}\} = N_1^* = 2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 = 160\,626\,866\,400$. The elements of $\mathbf{U_1}$ are of very specific structure which allows for every $N^* \in \mathbf{U_1}$ to choose explicitly two positive u, v depending on N^* in such a way that $G(K(u)\,K(v)\,N^*) > G(N^*)$, where $K(x)$ stands for the product of all primes p, $x < p \le 2x$.

AMS 2010 Subject Classification: 11A25, 11N05, 11N37.

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The author is deeply grateful to Valeriy V. Kozlov for his interest to this research.

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Fold-up Derivatives of Set-Valued Functions: Its Definition and Applications

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We give a survey on fold-up derivatives, a notion which was introduced in Khmaladze (2007) and extended in Khmaladze and Weil (2014) to describe infinitesimal changes in a set-valued function. We summarise the geometric background and compare the fold-up derivatives with other notions of differentials of set-valued functions, namely, those of tangent cones of, e.g., Aubin and Frankowska (1990), differential inclusions of, e.g., Artstein (1995) and, a very obvious choice, of generalised functions on boundary of sets.

We discuss in more detail applications in statistics, in particular to the change-set problem of spatial statistics, and show how the notion of fold-up derivatives leads to the theory of testing statistical hypotheses about the change set. We formulate Poisson limit theorems for the log-likelihood ratio in two versions of this problem and present also the route to a central limit theorem.

The talk is based on joint work with John Einmahl (Tilburg) and Wolfgang Weil (Karlsruhe). The review paper was invited to the Annals of the Institute of Statistical Mathematics, and will appear in January 2018.

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[x] And about 50 more in the forthcoming paper.

Extrapolation, Singular Integrals, Applications to the BVPs

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The goal of our talk is to present the extrapolation results in nonstandard Banach function spaces defined both on quasi-metric measure spaces and product spaces.Based on these results we prove the boundedness of non-harmonic analysis operators and give various applications to the BVPs of analytic functions.

Harmonic Univalent Maps and Pseudo-Hyperbolic Distance

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The class of harmonic univalent functions shall be introduced together with the hyperbolic metric and the psudo-hyperbolic distance. Emphacis shall be on potential theoretic issues. With the help of the properties of pseudo-hyperbolic distance, many new results on area distortion, of some classes of orientation preserving harmonic mappings such as the class of harmonic univalent maps onto the disk, the class of orientation preserving Bloch and BMOH maps. Issues and problems shall be suggested.

Stefan Banach and Georgia (On the occasion of Stefan Banach's 125-th Birthday Anniversary)

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Five years ago I wrote [1]:

1. On Saturday, March 15, 1941 "the delegation of Lvov State University, lead by S. S. Banach, the famous mathematician, arrived in Tbilisi… Banach, the head of delegation said… Our delegation will stay in Georgia for five days… "
2. "Gori, March 19. The professors of Lvov University comrades Banach, Zarits'kyi, docent Braginets and lady-student Solyak arrived to Gori".

In the first part of my talk I'll give some further comments about S. Banach's visit to Georgia. The second part of the talk will deal with two problems posed by him. One of the problems is taken from W. Orlicz's (1903–1990) paper [2, p. 124], while the another one is [3, Problem 106, p. 188]. I'll follow mainly [4] with some supplements related to the contribution of my colleagues from Georgia.

Acknowledgement. This talk was supported by N. Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University.

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Convergence and Summability of Vilenkin–Fourier Series in the Martingale Hardy Spaces

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The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves. Such waves have already been used frequently in the theory of signal transmission, multiplexing, filtering, image enhancement, codic theory, digital signal processing and pattern recognition. The development of the theory of Vilenkin-Fourier series has been strongly influenced by the classical theory of trigonometric series. Because of this it is inevitable to compare results of Vilenkin series to those on trigonometric series. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, which studies orthonormal systems from the point of view of the structure of a topological group.

This lecture is devoted to review theory of martingale Hardy spaces. We present central theorem about atomic decomposition of these spaces when $0 < p \leq 1$. We also present how this result can be used to prove boundedness of some operators on the martingale Hardy spaces and give a brief review how it can be connected to the theory of almost everywhere convergence. Moreover we present new estimations of Vilenkin-Fourier coefficients and prove some new results concerning boundedness of maximal operators of partial sums. Moreover, we derive necessary and sufficient conditions for the modulus of continuity so that norm convergence of the partial sums is valid and present Hardy type inequalities for the partial sums with respect to the Vilenkin systems. We also do the similar investigation for the Fejér means. Furthermore, we investigate some Nörlund means but only in the case when their coefficients are monotone. Some well-know examples of Nörlund means are Fejér means, Cesàro means and Nörlund logarithmic means. In addition, we consider Riesz logarithmic means, which is not example of Nörlund means. It is also proved that these results are the best possible in a special sense. As applications both some well-known and new results are pointed out.

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On Universal Series of Functions

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In our presentation we consider universal series with respect to systems of measurable and almost everywhere finite functions and represent several properties of such series. It follows from these properties theorems concerning universal series with respect to orthonormal systems of functions. Some of these theorems are also discussed.

Some Deterministic and Stochastic Estimations for Measuring Efficiency of Algorithms

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Dedicated to the memory of my father

The computational complexity of an algorithm is traditionally measured for the worst and also for the average case. The worstcase estimation guarantees a certain worst case behavior of a given algorithm, although it might be rough, as for “most of the instances”, the algorithm may have a significantly better performance. The probabilistic average-case analysis pretends to derive an average performance of an algorithm, say, for an “average instance” of the problem in question. That instance may be far away from the average of the problem instances arising in a given real-life application, and then, an average case analysis would also provide a non-realistic estimation. We suggest that, in general, a wider use of the probabilistic models for a more accurate estimation of the algorithm efficiency is possible. For instance, the quality of the solutions delivered by an approximation algorithm may also be estimated in “average” probabilistic case. Such an approach would deal with the estimation of the quality of the solutions delivered by the algorithm for the most common (for a given application) problem instances. As we will illustrate, probabilistic modeling can also be used to derive an accurate time complexity performance measure, distinct from the traditional probabilistic average-case time complexity measure. Such an approach could, in particular, be useful when the traditional average-case estimation is still rough or is not possible at all.

Abstracts of Participants' Talks

მონაწილეთა მოხსენებების თეზისები

On the Nonexistence of Global Solutions of Cauchy Problem for a Class of System of Nonlinear Hyperbolic Equations with Positive Initial Energy

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We consider the following initial boundary value problem:

$$u_{itt} - \Delta u_i + u_i + u_{it} = f_i(u_1, u_2, u_3), \quad x \in R^n, \quad t > 0, \quad i = 1, 2, 3, \tag{1}$$

with initial conditions

$$u_i(0, x) = u_{i0}(x), \quad u_{it}(0, x) = u_{i1}(x), \quad x \in R^n, \quad i = 1, 2, 3, \tag{2}$$

where $f_i(u_1, u_2, u_3) = |u_1|^{p_{i1}} |u_2|^{p_{i2}} |u_3|^{p_{i3}} u_i$, $\rho_{ii} = p_i - 1$, $\rho_{ik} = p_k + 1$, $i, k = 1, 2, 3$, p_1, p_2, p_3 are real numbers. The goal of this paper is to investigate nonexistence of global solutions to the problem $(1), (2)$.

Assume that

$$n \geq 2, \quad p_j \geq 0, \quad j = 1, 2, 3, \tag{3}$$

and additionally

$$p_1 + p_2 + p_3 \leq \frac{2}{n-2} \quad \text{if } \ n \geq 3. \tag{4}$$

Introduce the following notation

$$\begin{aligned} E(t) = \sum_{j=1}^{3} \frac{p_j + 1}{2} \left[|u'_{jt}|^2 + \|u_j\|^2 + \int\limits_0^1 |u'_{jt} s, \cdot)|^2 \, ds \right] \\ + \int R^n |u_1(t, x)|^{p_1+1} \cdot |u_2(t, x)|^{p_2+1} |u_3(t, x)|^{p_3+1} \, dx, \end{aligned}$$

$$I(\phi_1, \phi_2, \phi_3) = \sum_{j=1}^{3} \frac{p_j + 1}{p_1 + p_2 + p_3 + 3} \|u\|^2 - \int R^n |\phi_1(x)|^{p_1+1} \cdot |\phi_2(x)|^{p_2+1} |\phi_3(x)|^{p_3+1} \, dx.$$

Theorem. *Let conditions* (3), (4) *be satisfied,* $u_{i0} \in H^1$ *and* $u_{i1} \in L_2(R^n)$, $i = 1, 2, 3$, *and in addition, assume that the following conditions are satisfied:*

$$E(0) > 0, \quad I(u_{10}, u_{20}, u_{30}) < 0,$$

$$\sum_{j=1}^{3}\langle u_{j0},u_{j1}\rangle \geq 0, \quad \sum_{j=1}^{3}\frac{p_j+1}{2}|u_{j0}|^2 > \frac{p_2+p_2+p_3+3}{p_2+p_2+p_3}E(0).$$

Then the solution of the Cauchy problem (1), (2) *blows up in finite time.*

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Solvability of a Boundary Value Problem for Second Order Elliptic Differential-Operator Equations

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In the separable Hilbert space H we study the solvability of the following boundary value problem for second order elliptic differential equation

$$L(\lambda,D)u := \lambda u(x) - u''(x) + Au(x) = f(x), \ \ x\in(0,1), \tag{1}$$

$$L_1(\lambda)u := u'(0) + \alpha\lambda u(1) = f_1, \tag{2}$$

$$L_2 u := u(0) = f_2, \tag{3}$$

where λ is a complex parameter.

Theorem. *Let the following conditions be fulfilled: A is a strongly positive operator in H; $\alpha \neq 0$ is some complex number. Then problem* (1), (2) *for $f \in L_p((0,1);H(A^{1/2}))$, $f_1 \in (H(A),H)_{\frac{1}{2}+\frac{1}{2p},p}$, $f_2 \in (H(A^2),H)_{\frac{1}{4}+\frac{1}{4p},p}$, where $1<p<\infty$, and $|\arg\lambda| \leq \pi < \varphi$, $|\lambda|$ is sufficiently large, has a unique solution that belongs to the space $W_p^2((0,1);H(A),H)$ and, for these λ, the following noncoercive estimate holds for the solution of problem* (1), (2)*:*

$$
\begin{aligned}
|\lambda|\,\|u\|_{L_p((0,1);H)} + \|u''\|_{L_p((0,1);H)} + \|Au\|_{L_p((0,1);H)} \\
\le C\Big(|\lambda|\|f\|_{L_p((0,1);H(A^{\frac{1}{2}}))} + \|f_1\|_{(H(A),H)_{\frac{1}{2}+\frac{1}{2p},p}} \\
+ \|f_2\|_{(H(A^2),H)_{\frac{1}{4}+\frac{1}{4p},p}} + \|f_1\|_H + |\lambda|^{\frac{3}{2}-\frac{1}{2p}}\|f_2\|_H\Big).
\end{aligned}
$$

The boundary value problem similar to boundary value problem (1), (2) was studied in the paper [1].

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Global Bifurcation in Some Nonlinearizable Eigenvalue Problems with Indefinite Weight

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We consider the following fourth order boundary value problem

$$(p(t)u'')'' - (q(t)u')' = \lambda r(t)u + h(t,u,u',u'',u''',\lambda), \quad t \in (0,1), \tag{1}$$

$$
\begin{aligned}
u'(0)\cos\alpha - (pu'')(0)\sin\alpha = 0, \qquad u'(1)\cos\gamma + (pu'')(1)\sin\gamma = 0, \\
u(0)\cos\beta + Tu(0)\sin\beta = 0, \qquad u(1)\cos\delta - Tu(1)\sin\delta = 0,
\end{aligned} \tag{2}
$$

where $\lambda \in \mathbb{R}$ is a spectral parameter, $Ty \equiv (pu'')' - qu'$, the function $p(t)$ is strictly positive and continuous on $[0,1]$, $p(t)$ has an absolutely continuous derivative on $[0,1]$, $q(t)$ is nonnegative and absolutely continuous on $[0,1]$, the weight function $r(t)$ is sign-changing continuous on $[0,1]$ and α, β, γ, δ are real constants such that $0 \le \alpha, \beta, \gamma, \delta \le \pi/2$. The nonlinear term has the representation $h = f + g$, where $f, g \in C([0,1]\times\mathbb{R}^5)$ are real-valued functions satisfying the following conditions:

$$uf(t,u,s,v,w,\lambda) \le 0, \quad (t,u,s,v,w,\lambda) \in [0,1]\times\mathbb{R}^5,$$

there exists constants $M > 0$ such that

$$\Big|\frac{f(t,u,s,v,w,\lambda)}{u}\Big| \leq M, \quad (t,u,s,v,w,\lambda) \in [0,1] \times \mathbb{R}^5, \tag{3}$$

and

$$g(t,u,s,v,w,\lambda) = o\big(|u| + |s| + |v| + |w|\big) \tag{4}$$

in a neighborhood of $(u,s,v,w) = (0,0,0,0)$ uniformly in $t \in [0,1]$ and in $\lambda \in \Lambda$, for every bounded interval $\Lambda \subset \mathbb{R}$.

The problem (1), (2) with $r > 0$ under conditions (3) and (4) was considered in a paper [1] (see also [2]). In the case when r is a sign-changing function the methods of this work are not applicable.

Although problem (1), (2) is not linearizable in a neighborhood of the origin (when $f \not\equiv 0$), it is nevertheless related to a linear problem which is perturbation of problem (1), (2) with $h \equiv 0$. We estimate the distance between the principal eigenvalues of the perturbed and unperturbed problem. Using this estimation we find the bifurcation intervals. We show the existence of two pair of unbounded continua of solutions emanating from the bifurcation intervals and contained in the classes of positive and negative functions.

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Spectral Properties for the Equation of Vibrating Rod on Right End of which an Inertial Load is Concentrated

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We consider the following eigenvalue problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda y(x), \quad 0 < x < 1, \tag{1}$$

$$y(0) = y'(0) = 0, \tag{2}$$

$$y''(1) - a_1 \lambda y'(1) = 0 \tag{3}$$

$$Ty(1) - a_2 \lambda y(1) = 0, \tag{4}$$

where $\lambda \in \mathbb{C}$ is spectral parameter, $Ty \equiv y''' - qy'$, $q(x)$ is positive and absolutely continuous function on $[0, 1]$, a_1 and a_2, are real constants, such that $a_1 > 0$, $a_2 > 0$.

The problem (1)–(4) arises when variables are separated in the dynamical boundary value problem describing bending vibrations of a homogeneous rod, in cross-sections of which the longitudinal force acts, the left end of which is fixed rigidly and on the right end an inertial mass is concentrated (see [1, Ch. 8, § 5]).

Note that the signs of the parameters a_1 and a_2 play an important role. If $a_1 > 0$ and $a_2 < 0$, then problem (1)–(4), can be treated as a spectral problem for a self-adjoint operator in the Hilbert space $H = L_2(0, 1) \oplus \mathbb{C}^2$. This case was considered in [2]. If $a_1 > 0$ and $a_2 > 0$, then this problem is equivalent to a spectral problem for the self-adjoint operator in the Pontryagin space $\Pi_1 = L_2(0, 1) \oplus \mathbb{C}^2$ with the corresponding inner product.

In the present work we study the location of the eigenvalues on the real axis, the structure of root subspaces, we obtain asymptotic formulas for eigenvalues and eigenfunctions, and using these asymptotic formulas we establish sufficient conditions for the subsystems of eigenfunctions of (1)–(4) to form a basis in the space $L_p(0, 1)$, $1 < p < \infty$.

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A Collocation Method for Solving Fractional Bratu-Type Equation

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In this study, we obtain the numerical solution of the fractional Bratu-type equation using the sinc-collocation method. Numerical examples are presented to verify the efficiency and accuracy of the proposed algorithm. Finally, the approximate solutions are shown both graphically and in tabular forms.

Solving a System of Nonlinear Fractional Integro-Differential Equations

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In this study, we obtain approximate solutions of a system of nonlinear fractional integro-differential equations by the sinc collocation method. To demonstrate the effectiveness of the method, we give some examples. Also, we compare the obtained numerical solutions and their exact solutions by using Mathematica.

Nilpotent Exponential MR-Groups

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The notion of exponential R-group (R is an arbitrary associative ring with identity) was introduced by Lyndon in [1]. In [2], Myasnikov refined the notion of a exponential R-group by introducing and additional axiom. In particular the new notion of a exponential R-group is a direct generalization of the notion of a R-module to the case of non-commutative groups. In honour to Myasnikov, R-groups with this axiom in Amaglobeli's paper [3] has nomed MR-groups. This report is dedicated to the theory of varieties of nilpotent MR-groups. Moreover, the various analogs of the notion of n-class nilpotent group in the category of MR-groups are introduced, and it is shown that they coincide for $n = 1, 2$. The question whether these notions coincide remains open for $n > 2$.

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დისკუსია ტესტური გამოცდების შესახებ

ამირან ამბროლაძე

თბილისის თავისუფალი უნივერსიტეტი, მათემატიკისა და კომპიუტერული მეცნიერების სკოლა, თბილისი, საქართველო

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ამ მოხსენების მიზანია ინიცირება გაუკეთოს დისკუსიას გამოცდების ჩატარების ტესტური ფორმის შესახებ. კერძოდ:

- რა არის ტესტური გამოცდების პლუსები და მინუსები (ეს კითხვა დაკავშირებულია იმ საკითხთანაც, თუ რამდენად ხშირად უნდა გამოიცადნონ სტუდენტები სემესტრის განმავლობაში? საჭიროა თუ არა აქ ქართული რეალობის გათვალისწინებაც?)
- რა ფორმით ჯობს ჩატარდეს ასეთი გამოცდები (ტესტური გამოცდების ფორმებიც, თავის მხრივ, მრავალფეროვანია)?
- როგორი უნდა იყოს შეფასების სისტემა? კერძოდ, უნდა იყოს თუ არა „დასჯადი“ პასუხის ალალბედზე არჩევა? უფრო პოზიტიურად (მაგრამ ეკვივალენტურად) რომ ჩამოვაყალიბოთ ეს კითხვა: უნდა შეფასდეს თუ არა უფრო მაღალი ქულით პასუხი „არ ვიცი“, ვიდრე არასწორი პასუხი?

On the H-Wellposedness of the Singular Cauchy Problem for Systems of Linear Generalized Differential Equations

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Let $I \subset R$ be an interval non-degenerate in the point, $t_0 \in I$, $I_{t_0} = I \setminus \{t_0\}$. Consider the singular Cauchy problem for linear system of generalized ordinary differential equations

$$dx = dA(t) \cdot x + df(t) \ \text{ for } \ t \in I_{t_0}, \tag{1}$$

$$\lim_{t\to t_0+}(H^{-1}(t)\,x(t))=0, \tag{2}$$

where $A\in \mathrm{BV}_{loc}(I_{t_0};R^{n\times n})$ and $f\in \mathrm{BV}_{loc}(I_{t_0};R^n)$, i.e. A and f are, respectively, matrix- and vector-functions with bounded variations components on the every closed interval from I_{t_0}; $H=diag(h_1,\dots,h_n)$ is an arbitrary diagonal matrix-functions with continuous diagonal elements $h_i: I_{t_0}\to]0,+\infty[$ $(i=1,\dots,n)$.

The singularity is considered in the sense that A or f maybe have non-bounded total variation on the whole interval I.

We assume that $\det(I_n\pm d_jA(t))\neq 0$ for $t\in I_{t_0}$ $(j=1,2)$, where I_n is the identity $n\times n$-matrix, $d_1A(t)=A(t)-A(t-)$, $d_2A(t)=A(t+)-A(t)$. A vector function $x\in \mathrm{BV}_{loc}([a,b[;R^n)$ is said to be a solution of the generalized system (1) if

$$x(t)-x(s)=\int\limits_s^t dA(\tau)\cdot x(\tau)+f(t)-f(s) \ \text{ for } \ s,t\in I_{t_0}, \ \ s<t<b,$$

where the integral is understand in the Kurzweil–Stieltjes integral sense (see [1]).

Let x_0 be the unique solution of the problem (1), (2).

Along with the system (1) consider the sequence of the systems

$$dx=dA_k(t)\cdot x+df_k(t) \ \text{ for } \ t\in I_{t_0} \ \ (k=1,2,\dots,), \tag{3}$$

where the matrix- and vector-functions A_k and f_k $(k=1,2,\dots)$ have the properties circumscribed above.

There are considered the question about the sufficient conditions guaranteing the unique solvability of the singular problem (3), (2) for any sufficiently large natural k and for so called the H-converge of its solutions x_k $(k=1,2,\dots)$ to x_0 as $k\to+\infty$, i.e.

$$\lim_{k\to+\infty}(H(t))(x_k(t)-x_0(t))=0 \ \text{ uniformly for } \ t\in I_{t_0}.$$

The analogous problem has been investigated in [2] for systems of ordinary differential equations.

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On Compatible Group Topologies on LCA Groups

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The topology on a LCA group G is the finest locally quasi-convex group topology which gives rise to the character group $G^\wedge$ while the Bohr topology $\sigma(G, G^\wedge)$ is the coarsest topology with this property. For compact abelian groups both topologies coincide. A locally quasi-convex group topology is called compatible with the original topology if it has the same character group.

In the talk concrete examples for compatible topologies on $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{Z}(p^\infty)$ and on infinite products of discrete abelian groups will be given. Further, some properties of the poset $\mathcal{C}(G)$ of all compatible group topologies on G will be presented. If G is an infinite product of discrete groups, then $|\mathcal{C}(G)| = 2^{2^{|G|}}$ is as big as possible, and $|\mathcal{C}(\mathbb{R})| = \mathfrak{c}$, $|\mathcal{C}(\mathbb{Z})| = \mathfrak{c}$, and $|\mathcal{C}(\mathbb{Z}(p^\infty))| = \mathfrak{c}$ hold.

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Analysing Drug Therapy on the Interaction Between Tumor and Immune Cells Based on Fractional Differential Equations, and Optimal Control Theory

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A primary motivation in producing any mathematical model is to describe a natural or artificial phenomenon by means of a model equation whose behavior is as close as possible to the original phenomenon. But this is often difficult, particularly when we are dealing with nonlinear behavior in natural complex phenomena such as the interaction between variety of cancers and immune cells. In this article, we have used fractional myelogenous leukemia blood cancer's cells against naive T-cell and effective T-cell cells of body. Using this model, we have studied the dynamic behavior describing the transaction between bodies' effective T cell, naive T cell and chronic myelogenous leukemia in one side and drug in other side. The most important feature of the equations with fractional order derivatives is their non-localization. We expect that our fractional differential equations model will be superior to its ordinary differential equations counterpart in facilitating understanding of the natural immune interactions to tumor and of the detrimental side-effects which chemotherapy may have on a patient's immune system. Using this system, we will study the optimized drug dose in chronic myelogenous leukemia treatment with two methods namely targeted therapy and broad cytotoxic therapy.

Even the drug dose is important for cancer specialists, the weakness of immunology system in cancer affected patients, may results in additional problems for their body. Our goal is to find the best treatment regimens that minimize the cancer cell count and the deleterious effect of the drugs for a given patint. We examine the optimal control setting analytically, and include numerical solutions to illustrate the optimal regimens under various assumptions.

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On Deviations Between Kernel Type Estimators of a Distribution Density in $p \geq 2$ Independent Samples

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Let $X^{(i)} = (X_1^{(i)}, \dots, X_{n_i}^{(i)})$, $i = 1, \dots, p$, be independent samples of size $n_1, n_2, \dots, n_p$, from $p \geq 2$ general populations with distribution densities $f_1(x), \dots, f_p(x)$. It is based on sample $X^{(i)}$, $i = 1, \dots, p$, checking two hypotheses: the homogeneity hypothesis $H_0 :$ $f_1(x) = \cdots = f_p(x)$ and the goodness-of-fit hypothesis $H_0' :\ f_1(x) = \cdots = f_p(x) = f_0(x)$, where $f_0(x)$ is the completely defined density function. The density $f_0(x)$ is unknown.

Here the tests are constructed for the hypotheses H_0 and H_0' against the sequence of close alternatives: $H_1 :\ f_i(x) = f_0(x) + \alpha(n_0)\varphi_i(x)$, $\alpha(n_0) \to 0$, $n_0 = \min(n_1, \dots, n_p) \to \infty$, $\int \varphi_i(x)\,dx = 0$, $i = 1, \dots, p$. The test for the hypotheses H_0 and H_0' based on the statistic

$$T(n_1, n_2, \dots, n_p) = \sum_{i=1}^{p} N_i \int \Big[\widehat{f}_i(x) - \frac{1}{N}\sum_{j=1}^{p} N_j \widehat{f}_j(x)\Big]^2 r(x)\,dx, \tag{1}$$

where $\widehat{f}_i(x)$ is a kernel-type Rosenblatt–Parzen estimator of the density of the function $f_i(x)$:

$$\widehat{f}_i(x) = \frac{a_i}{n_i}\sum_{j=1}^{n_i} K\big(a_i(x - X_j^{(i)})\big), \quad N_i = \frac{n_i}{a_i}\,, \quad N = N_1 + \cdots + N_p.$$

We consider the question concerning the limiting law of the distribution of statistic (1) for the hypothesis H_1 when n_i tends to infinity so that $n_i = nk_i$, where $n \to \infty$, and k_i are constants. Let $a_1 = a_2 = \cdots = a_p = a_n$, where $a_n \to \infty$ as $n \to \infty$.

(i) $K(x) \geq 0$, vanishes outside the finite interval $(-A, A)$ and, together with its derivatives, is continuous on this interval or absolutely continuous on $(-\infty, \infty)$, $x^2 K(x)$ is integrable and $K^{(1)}(x) \in L_1(-\infty, \infty)$. In both cases $\int K(x)\,dx = 1$.

(ii) The density function $f_0(x)$ is bounded and positive on $(-\infty, \infty)$ or it is bounded and positive in some finite interval $[c, d]$. Besides, in the domain of positivity it has a bounded derivative.

(iii) Functions $\varphi_j(x)$, $j = 1, \dots, p$, are bounded and have bounded derivatives of first order; also $\varphi_i(x) \in L_1(-\infty, \infty)$.

(iv) The weight function $r(x)$ is piecewise-continuous, bounded and $r(x) \in L_1(-\infty, \infty)$.

Theorem. *Let the conditions* (i)–(iv) *be fulfilled. If* $\alpha_n = n^{-1/2} a_n^{1/4}$ $(\alpha_n = \alpha(n_0))$, $n^{-1} a_n^{9/2} \to 0$ *as* $n \to \infty$, *then for the hypothesis* H_1 *the random variable* $a_n^{1/2}(T_n - \mu)$ *has the normal limiting distribution* $(A(\varphi), \sigma^2)$, *where*

$$A(\varphi) = \sum_{i=1}^{p} k_i \int \Big[\varphi_i(x) - \frac{1}{\overline{k}} \sum_{j=1}^{p} k_j \varphi_j(x)\Big]^2 r(x)\, dx,$$

$$\sigma^2 = 2(p-1) \int f_0^2(x) r^2(x)\, dx \cdot R(K_0),\ K_0 = K * K,\ \mu = (p-1) \int f_0(x) r(x)\, dx \cdot R(K),$$

$$R(g) = \int g^2(x)\, dx, \quad \overline{k} = k_1 + \cdots + k_p,\ \ p \geq 2.$$

Inquiry-Based Teaching in Mathematics

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In the present talk is considered inquiry-based teaching in mathematics. Is considered the concept and basic steps for inquiry-based teaching in mathematics. In the given case is considered the using of electronic tools as research resource. These resources are: GeoGebra, Desmos/calculator, Microsoft Mathematics and etc. Also are considered particular specific examples in mathematics which leads to the inquiry. Also are considered examples of project based and problem based teaching in mathematics.

Non-Linear Versions of Characteristic Problems for the Second Order PDEs of Mixed Type

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We consider some specific second order PDEs. These equations are of hyperbolic type, but may degenerate under some known conditions for the derivatives of unknown solution. Our previous results include the construction of general integrals for these equations. Based on those results, we consider a non-linear version of Goursat characteristic problem ([1], [2]). In the case of this Characteristic problem, the sufficient conditions for existence and uniqueness of the solution are established. There is also defined a domain of definition of the solution. Another characteristic problem we investigated belongs to a class of so called non-local characteristic problems ([3]). The existence and uniqueness of a regular solution is proved also in this case.

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On Addition Formulas Related to Elliptic Genera

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We provide certain addition formulas for the logarithm of the universal Buchstaber formal group law and for the general elliptic integrals with four parameters having a differential $dt/dR(t)$, where $R(t)$ is a polynomial of degree 4. Our results specialize in Euler's addition theorems for elliptic integrals of the first and second kind.

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On Axiomatic Characterization of Bifunctor Homology Theory in the sense of Hu

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Bifunctor homology theory first axiomatized by H. Inassaridze [6] in terms of relative homology groups, the induced homomorphisms and the boundary homomorphisms.

In the paper we define bifunctor homology theory in the sense of Hu on the category **Comp** of compact Hausdorff spaces with coefficient in the abelian category **G**. G. Chogoshvili projective homology theory [3] with coefficient in the category of abelian groups in the bifunctor homology theory.

The aim of the paper is to construct axiomatic theory of bifunctor homology theory in the sense of Hu without using relative homology groups. Our axiomatic characterization is simpler and instead of Eilenberg–Steenrod axioms [5] uses the Hu's axioms formulated in terms of absolute groups, the induced homomorphisms, and the suspension isomorphism.

The main result of paper is following

Theorem 1. *There exists one and only one exact bifunctor homology theory on the category* **Comp** *with the coefficients in the category of the abelian groups witch satisfies the* *axioms of homotopy, suspension, dimension, and continuity for every infinitely divisible groups.*

The proof of this theorem is based on results of [6] and ([1], [4]).

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Initiating a New Trend in Complex Equations Studying Solutions in a Given Domain: Problems, Approaches, Results

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There is a huge number of investigations in complex differential equations when the solutions are meromorphic in the complex plane. The main attention was paid to the value distribution type phenomena of the solutions, particularly to the zeros (more generally to the a-points) of these solutions.

Meantime there are very few studies of meromorphic solutions in a given domain, particularly zeros of similar solutions weren't touched at all.

We initiate studies of meromorphic solutions in a given complex domain: we pose some new problems and give some approaches for their solutions.

Clearly for similar studies we should have some tolls that are valid for large classes of functions in a domain. As some tools we make use results related to three comparatively recent topics, Gamma-lines, proximity property and universal version of value distribution theory; all they are valid for any meromorphic function in a given domain.

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The Influence of the Inclusion on the Buckling Plate

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The paper deals with the problem of the local buckling caused by uniaxial stretching of an infinite plate with a circular inclusion from a different material. The effect of elastic modulus of the inclusion on the value of the critical load is investigated. In order to find the first critical load a variational principle is applied. The comparison of numerical results that were obtained in the Maple 18 and the results obtained by the finite element method in the ANSYS 13.1. The influence is analysed the ratio of the elastic properties of the inclusion and plate on the value of the critical load and the form of the loss of stability.

Suppose that: E_1, ν_1 – are Young's modulus and Poison's ratio of the plate, and E_2, ν_2 – parameters of the inclusion. R – radius of the inclusion, and x, y Cartesian coordinates. The plate is presented in Figure 1.

The stresses act along the y axis. Let us denote the stress field inside the inclusion as $\sigma_{xx} = k_x\sigma$ and $\sigma_{yy} = k_y\sigma$, where k_x and k_y are the coefficients which were defined in [1] and these coefficients are responsible for the relationship between the elastic modulus of the insert and the plate.

The first critical load that causes the loss of stability can be evaluated with the use of the energy method customized by Timoshenko and Ritz. Figure 2 shows the dependence of the critical load on the ratio between the modulus of the inclusion and plate (σ_0 is the critical load corresponding to the plate with a hole of radius R). Calculations show that the loss of stability of a plate with a circular rigidly fixed inclusion happens at lower loads when the modulus of elasticity of the inclusion is either much smaller than the plate (i.e. the inclusion is very "soft") or, conversely, much larger (i.e., inclusion very "tough").

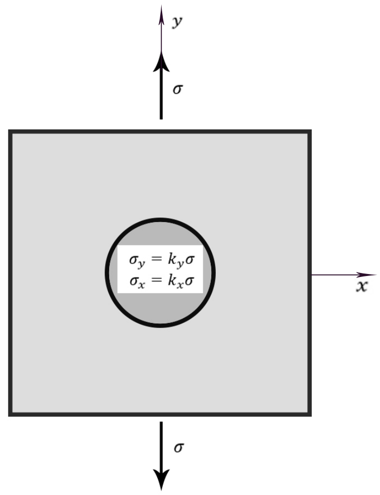

Figure 1. Plate with a Circular Inclusion Under Tensile Stress

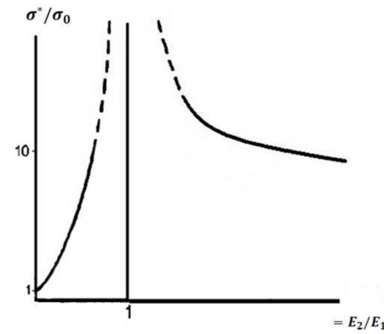

Figure 2. Dependence of the critical load on the ratio of the inclusion module to the plate module

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Soft Countable Topological Spaces

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In 1999, Russian researcher Molodtsov proposed the new concept of a soft set which can be considered as a new mathematical approach for vagueness. Soft topological spaces have been studied by some authors in recent years.

The purpose of this paper is to investigate some concepts of soft topological spaces. We give some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft separable spaces, soft sequential continuity and some important properties of these concepts are investigated.

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Questions of Existence of Periodic Solutions for Functional-Differential Equations of Pointwise Type

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The report is devoted to the existence of periodic solutions for ordinary differential equations, as well as for functional differential equations of pointwise type with quasilinear right sides. The developed approach is based on taking into account the asymptotic properties of the solutions of differential equations that were not taken into account in the study of periodic solutions, since we considered restrictions of such equations to an interval equal to the period. Such conditions for the existence of a periodic solution obtained on the basis of the study of the action of the shift operator, but taking into account its asymptotic properties, are new even for ordinary differential equations and essentially expand the class of non-autonomous ordinary differential equations for which this approach is applicable.

Triangulation of Polyhedra and Their Applications

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In our presentation we consider one of the most important topic of combinatorial geometry – triangulation of polyhedrons. It is well known that triangulation have a many applications in various branch of mathematics and not only in mathematics but also in other fields of sciences such, that Geodesy, Bibliography, Research of Design, Modern Technology, Engineering, Navigation, Meteorology, Astrometry, Developing of Arms and so on. Algorithmic study of triangulations was founded in the 70th years in XX century (see [1], [2]).

The goal of presented topic is study the various problems connected to the triangulation of polyhedrons in three dimensional Euclidean space, and their applications in the various fields of mathematics, in particular in the study of mathematical education. More precisely, in $\mathbf{R}^2$ plane it is workable (possible) to calculate the number of triangulation of polygons, but similar question in $\mathbf{R}^3$ space is more difficult (see [3]).

In particular, in three dimensional Euclidean space during triangulation of convex polyhedron we lose some invariants, such that number of diagonals, number of tetrahedrons and so on. The presentation is an interdisciplinary topic. We investigate application of algebraic or geometrically structures, algorithms and methodology of theory of graphs to computational geometry.

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On the Axiomatic Systems of Steenrod Homology Theory of Compact Spaces

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The Steenrod homology theory on the category of compact metric pairs was axiomatically described by J.Milnor. In [6] the uniqueness theorem is proved using the Eilenberg-Steenrod axioms and as well as relative homeomorphism and clusres axioms. J. Milnor constructed the homology theory on the category $\mathbf{Top^2_C}$ of compact Hausdorff pairs and proved that on the given category it satisfies nine axioms - the Eilenberg-Steenrod, relative homeomorphis and cluster axioms (see Theorem 5 in [6]). Besides, he proved that constructed homology theory satisfies partial continuity property on the subcategory $\mathbf{Top^2_{CM}}$ (see Theorem 4 in [6]) and the universal coefficient formula on the category $\mathbf{Top^2_C}$ (see Lemma 5 in [6]). On the category of compact Hausdorff pairs, different axiomatic systems were proposed by N. Berikashvili [1], [2], H. Inassaridze and L. Mdzinarishvili [4], L. Mdzinarishvili [5] and H.Inasaridze [3], but there was not studied any connection between them. The paper studies this very problem. In particular, in the paper it is proved that any homology theory in Inassaridze sense is the homology theory in the Berikashvili sense, which itself is the homology theory in the Mdzinarishvili sense. On the other hand, it is shown that if a homology theory in the Mdzinarishvili sense is exact functor of the second argument, then it is the homology in the Inassaridze sense.

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ჰემიტროპული დრეკადობის თეორიის სამგანზომილებიანი დინამიკის ამოცანის ამოხსნა ფურიეს მეთოდით

იური ბეჟუაშვილი

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შესწავლილია სამგანზომილებიანი დინამიკური ამოცანა ერთგვაროვანი ჰემიტროპული მიკროპოლარული სხეულისათვის. ფურიეს მეთოდის გამოყენებით დამტკიცებულია კლასიკური ამოხსნების არსებობისა და ერთადერთობის თეორემები.

Completely ⊕-Supplemented Lattices

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In this work, all lattices are complete modular lattices. Let L be a lattice. If every quotient sublattice $a/0$ such that a is a direct summand of L is $\oplus$-supplemented, then L is called a completely $\oplus$-supplemented lattice. In this work, some properties of these lattices are investigated.

Proposition 1. *Every complemented lattice is completely $\oplus$-supplemented.*

Corollary 2. *Let L be a supplemented lattice with $r(L) = 0$. Then L is completely $\oplus$-supplemented.*

Proposition 3. *Let L be a lattice with $(D1)$ property. Then L is completely $\oplus$-supplemented.*

Proposition 4. *Let L be a lattice with $(D3)$ property. Then L is completely $\oplus$-supplemented.*

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An Ontology Model for a Tourism Web Portal

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In the talk we present an ontology model of a semantic web portal, based on the Description Logics. A semantic web portal is a web page, based on semantic web technologies, where human readable information is accompanied by machine readable information, usually written in ontologies. The topic of our web portal is tourism in Georgia, where user can automatically plan trips (the places to visit in Georgia) based on his budget, length of visit and some other criteria. To the best of our knowledge, there is no such automated system exists so far.

On the Solutions of Elastic Materials with Voids

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In this paper the 2D quasi-static theory of elasticity for materials with voids is considered. The representation of regular solution of the system of equations in the considered theory is obtained. There the fundamental and some other matrixes of singular solutions are constructed in terms of elementary functions.

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The Cauchy Problem for the Equation Describing Hydrodynamic Processes in Magnetohydraulic Pusher

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The work proposes the initial Cauchy problem for the equation describing hydrodynamic processes in the magnetohydraulic pusher of an original construction. There is shown the uniqueness of solution, which is written in the explicit form and its domain of propagation is established.

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The System of Mathematical Tasks Solving by Using the Method of Area

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Different kinds of tasks are given in the modern books. During their solving less attention is paid to the psychological factors affecting mathematics. In this case, we offer the solution of the mathematical tasks by using the method of area.

Osculating Directional Curves of Non-Lightlike Curves in Minkowski 3-Space

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In this study, we introduce the notion of osculating directional curve and osculating donor curve of the non-lightlike Frenet curve in the Minkowski 3-space E_1^3 and give some characterizations and results for these curves.

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Computer Modeling of Process of Two-Level Assimilation

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In work numerical results of computer modeling of earlier offered mathematical model of process of two-level assimilation in case of constancy of all parameters of model are given [1]–[3].

In a numerical experiment demographic factor of the first side is negative, the third – is positive, and demographic factor of the second side is taken both positive and negative. The numerical experiment shows that required functions have periodic character, at the same time the function defining the third side which is under double assimilation accepts positive values.

As a result of computer modeling of process of two-level assimilation it is shown that if indicators of demographic factor, assimilation coefficients, initial values of required functions are taken from some intervals, then there is no full assimilation of the third side (state formation, autonomy).

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Mathematical Model of Information Warfare with System of Linear Partial Differential Equations

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In work in development of earlier considered model of information warfare with participation two confronting and the third peacekeeping the sides [1]–[3], with flows of information depending only on time the new mathematical model of information warfare, where flows of information of all three sides depend both on a time and on the second "geometrical" variable characterizing the level of development of technology of information transfer is offered.

The mathematical model is described by an initial-boundary task for system of three linear differential equations in partial derivatives, and the task is considered in some closed rectangle of the first quarter of the plane of two independent variables, and on the left end of a segment of change of a "geometrical" variable homogeneous conditions of Dirichlet are given.

In some special case of constants of model, for the third peacekeeping side, the parabolic differential equation in partial derivatives of the second order with the corresponding initial and boundary conditions is received. Some exact decisions are received.

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Nonlinear Mathematical Model of Transformation of Two-Party Elections to Three-Party Elections

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In work the new nonlinear mathematical model of transformation of two-party elections to three-party elections is offered. In model it is supposed that two parties participated in the previous elections and as a result of elections one of parties became pro-governmental and the second oppositional. Before the following elections to political arena there is the second opposition party and on the subsequent elections two oppositional and one pro-governmen-tal parties already participate.

In that special case, when demographic factor of elections for all three parties is equal to zero, coefficients of involvement of voters are constant, but are excellent during the periods of two-party membership and three-party membership, Cauchy's task in the first interval for system of two, and in the second interval of three nonlinear differential equations is solved analytically exactly. Taking into account indicators of appearances at the next elections of all three parties supporting voters, certain falsifications of voices of opposition parties, necessary conditions for a victory of pro-government party are found. The found exact analytical solutions allow all three parties to select strategy for a win of elections.

Mathematical Model of Interference of Fundamental and Applied Researches

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In work the new nonlinear continuous mathematical model of interference of fundamental and applied researches of scientifically-research institute (micro-model) is considered. For a special case, Cauchy's problem for nonlinear system of differential equations of first order is definitely decided analytically. In more general case based on Bendikson's criteria the theorem of not existence in the first quarter of the phase plane of solutions of closed integral curves is proved. Conditions on model parameters in case of which existence of limited solutions of system of nonlinear differential equations is possible are found.

The offered nonlinear mathematical model allows to estimate influence of fundamental and applied researches (works) at each other, to find conditions on constant models in case of which there can be limited solutions, i.e. closed integral curves in the first quadrant of the phase plane of solutions.

Nonlinear Two or Three-Stage Mathematical Model of Training of Scientists

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In work it is offered two or three-stage nonlinear mathematical model of training of research associates (scientists). In two-level model two subjects are considered: research associates without degree and the diplomaed scientists with doctor's degree, and in three-stage model – three subjects: research associates without degree, candidates of science and the diplomaed scientists with doctor's degree. For system of two or three nonlinear differential equations Cauchy's task is set. The mathematical model describes process of self-reproduction of research associates (preparation), their irreversible exit or transition from one category to another.

The two-level model actually comes down to the known model "the victim" (research associates without degree) – "a predator" (scientists with doctor's degree) taking into account the intraspecific competition (members with self-limitation of increase). The system of the nonlinear differential equations in the first closed quarter of the phase plane of decisions has three provisions of balance, and the balance position corresponding to the trivial decision is a saddle at any values of parameters of model, the second position of balance corresponding to extinction of "predators" and to an equilibrium condition of "the victims" in one case is a saddle, and in the second–steady knot.

Conditions on constant models for which the stationary decision, the third position of balance in an open first quarter of the phase plane of decisions (the only limit point of system of the differential equations) corresponding to equilibrium coexistence of "predators" and "the victims" asymptotically is steady (steady knot or steady focus) are found.

Lack of periodic trajectories of system of the nonlinear differential equations is proved.

Simulation and Analysis of Some Non-Ordinary Atmosphere Processes by WRF Model Based on the GRID Technologies

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Regional scale summer time precipitations, among others (wind, temperature, sea level pressure, geopotential height etc.), still remain more difficult parameter for prediction by weather research forecast (WRF) model. This inconvenience mainly is stipulated by insufficient parameterization the resolved and subgrid-scale precipitation processes in the WRF model and by the lack in setting reliable initial and boundary conditions at nested grids of the WRF model. Furthermore, in comparison with cold-season precipitations, warm-season convective events and precipitations are much more difficult for prediction and especially for the territories having complex orography. In this study the problem of micro physics and cumulus parameterization schemes options for several warm-season convective events predictions above the Caucasus territory is studied. With the purpose of investigating impact of detailed orography on summer time heavy showers prediction three set of domains with horizontal grid-point resolutions of 19.8, 6.6 and 2.2 km have been used. Computations were performed by Grid system with working nodes (16 cores+, 32GB RAM on each) situated at GE-01-GRENA. Some results of the numerical calculations performed by WRFv.3.6.1 model with different microphysics and convective scheme components are presented.

Acknowledgment. The research leading to these results has been co-funded by the European Commission under the H2020 Research Infrastructures contract no. 675121 (project VI-SEEM).

About Nonlocal Contact Problems

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The present work is devoted to the review of the articles, where for some equations of the mathematical physics the boundary and initial-boundary problems with nonlocal contact conditions are considered. For these problems, the existence and uniqueness of the solution is proved. The algorithms for numerical solution are constructed and investigated.

Some Issues of Data Sufficiency about Solving the Problem

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While studying math in school mathematics courses, the greatest attention is given to the task solving. The practice proved that most of the graduates of the school find it difficult to solve unknown tasks. Moreover, they are completely powerless in front of it. So that we think that quite more attention must be given to the task solving about data sufficiency. If the pupil is well aware of which condition is appropriate for the task to be solved and which is not, the content of the task is clear and the mathematical model of a verbal formulated task is created, it no longer be difficult. The problem is that pupils cannot understand the content of the text. In this paper, three types of task solving models about data sufficiency are proposed.

Some Issues of Teaching Division with Remainder in School Math Course

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Present work deals with the clear interpretation of the concept of division with remainders in the school math course and its relation with life problems. It is noteworthy that in certain books and on the internet we come across with incorrect clear interpretation of problems related to the division with a remainder. For a student this obstructs the correct consideration of the most serious math issue - division with remainder. The work presents analysis of the division with remainders in school math course, hence, its proper clear interpretation, which is very important to correctly consider the division with remainders.

The Boundary Value Problem for Some Class of Second Order Hyperbolic Systems

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Darboux type problem with Dirichlet and Poincaré boundary conditions for one class of nonlinear second order hyperbolic systems is considered. The questions of existence and nonexistence, uniqueness and smoothness of global solution of this problem are investigated.

The case when the nonlinearity in the system is of an integral nature is considered also.

Scintillation Effects and the Spatial Power Spectrum of Scattered Radio Waves in the Ionospheric F Region

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Differential equation for two-dimensional spectral function of the phase fluctuation is derived using the modify smooth perturbation method. Second order statistical moments of the phase fluctuations are calculated taking into account polarization coefficients of both ordinary and extraordinary waves in the turbulent collision magnetized plasma and the diffraction effects. Analytical and numerical investigations in the ionospheric F region are based on the anisotropic Gaussian and power law spectral functions of electron density fluctuations including both the field-aligned anisotropy and field-perpendicular anisotropy of the plasma irregularities. Scintillation effects in this region are investigated for the small-scale ionospheric irregularities. Splashes caused by strong phase fluctuations are revealed in the normalized scintillation level at 40 MHz incident wave. Increasing frequency from 30 MHz up to 40 MHz scintillation level decreases for both spectra. Phase structure function is calculated for arbitrary correlation function of electron density fluctuations. Varying anisotropy factor the angle of arrival in the principal plane than in perpendicular plane. The large-scale background plasma structures are responsible for the "Double-Humped Effect" in the spatial power spectrum taking into account diffraction effects. This spectrum of scattered electromagnetic waves has two maximums if large scale plasma irregularities strongly elongated along the lines of forces of geomagnetic field. At fixed collision frequency the gap of the curve decreases two times increasing anisotropy factor four times. Estimations show that the this new effect exists even small scale plasma irregularities are oriented perpendicular to the external magnetic field. Numerical calculations are based on the experimental data of the navigation satellites.

On One Problem of Reduction

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The problem of optimal stopping with incomplete data is reduced to the problem of optimal stopping with complete data. Let us consider the following model of Kalman–Bucy scheme [1]:

$$\begin{gathered} d\theta_t = a(t)\theta_t dt + \epsilon_1 dw_1(t), \\ d\xi_t = d\theta_t + \epsilon_2 dw_2(t), \ \ 0 \le t \le T, \end{gathered}$$

where $\epsilon_1 > 0$, $\epsilon_2 > 0$ are constants, w_1 and w_2 are independent Wiener processes. It is assumed that θ is the nonobservable component and ξ is the observable one.

Let us define the following processes $m_t = E[\theta_t|\Im_t^\xi]$ and $\gamma_t = E(\theta_t - m_t)^2$. Let us introduce the payoff function [2]:

$$S^{\epsilon_1,\epsilon_2} = \sup_{\tau\in\Re^\xi} E\theta_\tau,$$

where supremum is taken over the class of stopping times $\Re^\theta$ (respectively $\Re^\xi$) with respect to the filtration $\Im_t^\theta = \sigma\{\theta_s: \ 0 \le s \le t\}$ (respectively $\Im_t^\xi = \sigma\{\xi_s: \ 0 \le s \le t\}$).

Theorem 1. For the payoff $S^{\epsilon_1,\epsilon_2}$ we have

$$S^{\epsilon_1,\epsilon_2} = \sup_{\tau\in\Re^\xi} Em_\tau.$$

Theorem 2. *The following representation is valid payoff $S^{\epsilon_1,\epsilon_2}$ we have*

$$S^{\epsilon_1,\epsilon_2} = \sup_{\tau\in\Re^\theta} E\widetilde{\theta}_\tau,$$

where

$$\widetilde{\theta}_t = \Phi_t \Bigg[\int\limits_0^t \Phi_s^{-1} a(s)\, ds + \int\limits_0^t \Phi_s^{-1} \frac{a(s)\gamma_s}{\sqrt{\epsilon_1^2 + \epsilon_2^2}}\, dw_1(s) \Bigg], \quad \Phi_t = \exp\Bigg\{ \int\limits_0^t a(s)\, ds \Bigg\}.$$

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Solving Matching Equations in Variadic Equational Theories

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In this talk, we present our recent results about matching in variadic equational theories. In particular, we have designed a sound and complete matching procedure for these theories, computing minimal set of solutions. In general, the procedure is not terminating, since some problems might have an infinite set of solutions. We have identified matching fragments for which the procedure stops and returns a minimal and complete set of solutions. We have also restricted the procedure to a variant of matching, where solutions are of a certain shape, and proved its termination and minimality.

On the Summability of One-Dimensional Associated Fourier Series

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It is known that to each summable in the n -dimensional cube $[-\pi,\pi]^n$ function f of variables $x_1,\dots,x_n$ there corresponds one n-multiple trigonometric Fourier series $S[f]$ with constant coefficients.

With the function f are associate n one-dimensional Fouries series $S[f]_1,\dots,S[f]_n$ with respect to variables $x_1,\dots,x_n$, respectively, with nonconstant coefficients. If a continuous function f is differentiable at some point $x=(x_1,\dots,x_n)$, then all one-dimensional Fourier series $S[f]_1,\dots,S[f]_n$ converge at x to the value $f(x)$.

We consider a summability of associate series with f [1].

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On Teaching of Mathematics to the Students of Natural Sciences

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We have prepared textbook "Mathematics for the Students of Natural Sciences" for the second edition. The first edition (Innovation, Tbilisi, 2006) has been revised and filled with some themes and exercises. The textbook includes the elements of Analytical Geometry, Linear and Vector Algebra, Differential and Integral Calculus. All the basic issues have examples and exercises. Some simple mathematical models of chemical, biological, ecological, medical and other processes are considered.

On the H-Wellposedness of the Singular Cauchy Problem for Systems of Linear Impulsive Differential Equations

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Let $I \subset R$ be an interval non-degenerate in the point, $t_0 \in I$, $I_{t_0} = I \setminus \{t_0\}$. Consider the linear system of impulsive equations with fixed points of impulses actions

$$\frac{dx}{dt} = P(t)\,x + q(t) \ \text{ for a.a. } \ t \in I_{t_0} \setminus \{\tau_l\}_{l=1}^{+\infty}, \tag{1}$$

$$x(\tau_l+) - x(\tau_l-) = G_l\,x(\tau_l) + g_l \ \ (l = 1, 2, \dots); \tag{2}$$

$$\lim_{t\to t_0+} (H^{-1}(t)\,x(t)) = 0, \tag{3}$$

where $P \in L_{loc}(I_{t_0}, R^{n\times n})$, $q \in L_{loc}(I_{t_0}, R^n)$, i.e. P and q are, respectively, matrix- and vector-functions with integrable components on the every closed interval from I_{t_0}; $G_l \in R^{n\times n}$ $(l = 1, \dots, m)$, $g_l \in R^n$ $(l = 1, \dots, m)$, $\tau_i \neq \tau_j$ if $i \neq j$, and $t_0 \notin \{\tau_1, \tau_2, \dots\}$; $H = diag(h_1, \dots, h_n)$ is an arbitrary diagonal matrix-functions with continuous diagonal elements $h_i : I_{t_0} \to]0, +\infty[$ $(i = 1, \dots, n)$.

The singularity of system (1), (2) is considered in the sense that the matrix P and vector q functions, in general, are not integrable at the point t_0.

We assume that

$$\det(I_n + G_l) \neq 0 \ \ (l = 1, 2, \dots),$$

where I_n is the identity $n \times n$-matrix.

Let x_0 be the unique solution of the problem (1), (2); (3).

Along with the system (1), (2) consider the sequence of the systems

$$\frac{dx}{dt} = P_k(t)\,x + q_k(t) \ \text{ for a.a. } \ t \in I_{t_0} \setminus \{\tau_l\}_{l=1}^{+\infty}, \tag{4}$$

$$x(\tau_l+) - x(\tau_l-) = G_{kl}\,x(\tau_l) + g_{kl} \ \ (l = 1, 2, \dots); \tag{5}$$

$(k = 1, 2, \dots)$, where P_k, G_{kl}, q_k and g_{kl} $(k, l = 1, 2, \dots)$ have the properties circumscribed above.

We consider the question on the sufficient conditions guaranteing the unique solvability of the singular problem (4), (5); (3) for any sufficiently large natural k and for so called the H-converge of its solutions x_k $(k = 1, 2, \dots)$ to x_0 as $k \to +\infty$, i.e.

$$\lim_{k\to+\infty} (H(t))(x_k(t) - x_0(t)) = 0 \text{ uniformly for } t \in I_{t_0}.$$

The analogous problem has been investigated in [1] for systems of ordinary differential equations.

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Optimal Variational Iteration Approximation of a Fourth-Order Boundary Value Problem

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In this study, we present a semi-analytical solution of a nonlinear fourth-order integro differential equation subject to two point boundary conditions arising in the study of transverse vibrations of a hinged beam by Optimal Variational Iteration Method. The most recently established methodology, which involves an auxiliary parameter and an auxiliary linear differential operator, is the approach used in this study in order to achieve efficient numerical solution. A sufficient way is considered for determining an optimal value of auxiliary parameter by the way of minimizing the residual error over the problem domain. The numerical examples and their residual error computations are given to support the accuracy and efficiency of the proposed technique.

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Measure Theoretical Approach for Solving 3-D Optimal Shape Design Problems in Spherical Coordinates

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In this paper, the problem of determining a bounded shape located over the (x, y)-plane, with a given projection area and volume such that it minimizes some given surface integral, has been examined. In order to determine the optimal shape, by regarding some useful advantages of spherical coordinates, we have extended a measure theoretical embedding process for shape optimization problems in spherical coordinates. First, the problem is converted into a measure theoretical space as an infinite dimensional linear optimization problem. Then it is relaxed into a finite dimensional linear programming by using approximation schemes. Finally, the solution of this LP is used to identify some points of the nearly optimal surface. Then, using an suitable outlier detection algorithm and a kind of MATLAB smooth curve fitting toolbox gives us the nearly optimal smooth surface based on the given optimal points. This approach in comparison to the other methods, has some important advantages: linear treatment for even strongly nonlinear problems, global optimization without requiring an initial shape and mesh designing.

მიზნობრივი ფუნქციების საზღვრების პოვნის სწრაფი ალგორითმები

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კონტეინერებში შეფუთვის ამოცანების კვლევა ჩვენი ჯგუფის მიერ უკვე რამდენიმე წელია გრძელდება. გასულ წელთან შედარებით შეიქმნა ამოცანათა კიდევ ხუთი, სულ დამუშავდა ოცდაათზე მეტი სხვადასხვა ამოცანის მოდელი და მათი ამოხსნის ე. წ. „სწრაფი ალგორითმები“, რომლებიც საშუალებას იძლევიან ნაპოვნი იქნას მიზნობრივი ფუნქციების საზღვრები და მოხდეს მათი ოპტიმალურობის შეფასება მინიმუმისა და მაქსიმუმის ფარგლებში.

ჩვენი მიზანი იყო მომხმარებლისათვის მიგვეწოდებინა მოსახერხებელი ინსტრუმენტი – პროგრამული პროდუქტი შეფუთვის მათი ამოცანების ამოსახსნელად. ინსტრუმენტი იძლევა რამდენიმე მნიშვნელოვან შედეგს, რომელთაგან გამოვყოფთ ორ მთავარს, ესენია: საუკეთესო მიახლოებითი ამონახსნისა და მიზნობრივი ფუნქციის საუკეთესო საზღვრის პოვნა, რომელთანაც ჩვენი ინსტრუმენტი გასცემს დროის მოცემული ლიმიტის ფარგლებში. მოძებნილი საზღვრებით შეგვიძლია მიახლოებითი ამონახსნების ხარისხის შეფასება, როგორც ოპტიმალურ ამონახსნთან ახლოს მდგომისა. პროგრამული პროდუქტი შედგება სამი ნაწილისაგან: 1. მომხმარებლის ინტერფეისი; 2. ამომხსნელი; 3. ინტერაქტიული შეფუთვა.

დამუშავებულია ინფორმაციული ინტერაქტიული კომპიუტერული სისტემა, რომელიც მრავალ მომხმარებელს საშუალებას მისცემს თავისი კონკრეტული საკითხის გადაწყვეტისათვის პასუხი მიიღოს საწყისი მონაცემების (ამოცანის პირობების) კომპიუტერში შეყვანით, რისთვისაც მათ არ დასჭირდებათ სპეციალური მათემატიკური აპარატის ცოდნა. ამავე დროს, სისტემას შეუძლია დიდი რაოდენობის ერთგანზომილებიანი ამოცანების მოდელები გააერთიანოს ერთ საერთო პროგრამაში და იპოვოს მიზნობრივი ფუნქციების საზღვრები, რათა მოხდეს მიახლოებითი ამოხსნების შეფასება.

გარდა ზემოთაღნიშნულისა, დამუშავებულია ამონახსნების შეფასების ორიგინალური ტექნოლოგია, რომლის გამოყენებით ხდება პრობლემური ამოცანების მიზნობრივი ფუნქციონალების შეფასება (მინიმუმის ამოცანებისათვის – ქვედა საზღვრების და მაქსიმუმის ამონაცებისათვის – ზედა საზღვრებისათვის).

დამუშავებულია 34 კომბინატორული მოდელის ამოხსნის დისტანციური პროგრამული ინსტრუმენტი, რომლებიც სემანტიკურად ახლოს არის კონტეინერებში შეფუთვის ერთგანზომილებიან ცნობილ ამოცანებთან. ყველა მოდელს გააჩნია სხვადასხვა სფეროში პრაქტიკული გამოყენება.

Simply Connected Nilmanifolds

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Let $\mathfrak{g}$ be a Lie algebra and G be the corresponding connected and simply connected Lie group. A metric Lie algebra $(\mathfrak{g}, \langle\cdot,\cdot\rangle)$ is a Lie algebra $\mathfrak{g}$ together with a Euclidean inner product $\langle\cdot,\cdot\rangle$ on $\mathfrak{g}$. This inner product on $\mathfrak{g}$ induces a left invariant Riemannian metric on the Lie group G. If $(\mathfrak{n}, \langle\cdot,\cdot\rangle)$ is a nilpotent metric Lie algebra, then the corresponding nilpotent Lie group N endowed with the left-invariant metric arising from $\langle\cdot,\cdot\rangle$ is a Riemannian nilmanifold. The isometry group of Riemannian nilmanifolds and the totally geodesic subalgebras of metric nilpotent Lie algebras are popular subjects for investigations (cf. [1]–[6]). Together with P. T. Nagy we classify the isometry equivalence classes and determine the isometry groups of Riemannian nilmanifolds on all five dimensional simply connected non two-step nilpotent Lie groups and on all simply connected standard filiform Lie groups. In this classification the metric Lie algebras which possess an orthogonal direct sum decomposition into one-dimensional subspaces play an important role.

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Dynamics of the Electron Spins $S = 1$ with the Zero-Field Level Splitting in the Molecular Crystals in a Strong Constant Field

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Some questions of the dynamics of the electron spins S=1 possessing the zero-field splitting are investigated in a strong constant magnetic field. The equations of motion of the magnetization components related to the separate transitions of the EPR fine structure (FS) are derived. It is demonstrated that the free motion of the sample magnetization is the precession at the frequency of the excited "allowed" transition with an ellipse in the plane transverse to the constant field, which is accompanied by the longitudinal magnetization component oscillation at the frequency of the "forbidden" transition. The tensor of the dynamic susceptibility of the spin-system (SS) to the microwave field is written at the creation of the resonance conditions for each transition of the well resolved FS. The matrix elements of this tensor reflect the elliptical character of the magnetization precession at the frequency of the very same transition, for which the resonance conditions are created. The SLR rates of the separate transitions of the FS are calculated at the same mechanism. The low-frequency method of their measurement is suggested with the help of the Gorter type experiment in a strong constant field transforming the three-level SS into the two-level one with the analytically derived expressions for the three non-zero diagonal elements of the low-frequency susceptibility tensor at the three directions of the constant field.

Boundary Contact Problems for Hemitropic Elastic Solids with Friction

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We investigate boundary-contact problem of statics of the theory of elasticity for homogeneous hemitropic elastic medium with regard friction. In this case on the part of the surface where friction occurs, instead of the normal component of the body force, a normal component of displacement is given. We consider two cases, the so-called coercive (when elastic media is fixed along some parts of boundary), and the semi-coercive case (the boundary is not fixed). By using the Steklov–Poincaré operator, we reduce this problem to an equivalently boundary variational inequality. Based on our variational inequality approach,we prove existence and uniqueness theorems for weak solutions. We prove that the solutions continuously depend on the data of the original problem. In the semi-coercive case, the necessary condition of solvability of the corresponding contact problem is written out explicitly. This condition under certain restrictions is sufficient as well.

The Solutions of Stochastic Differential Equations Connected with Nonlinear Elliptic Equations

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In the paper we consider the stochastic differential equations. The solutions of these equations connected with nonlinear elliptic equations are studied.

Key words: Stochastic differential equations, nonlinear equation, elliptic.

Numerical Model of a Mesoscale Boundary Layer of the Atmosphere and Some Processes Proceeding in It

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On the basis of developed by us two dimensional (in the vertical $x - z$ plane), non-stationary numerical model of the mesoscale boundary layer of the atmosphere (MBLA) the following works are carried out:

The "dry" model, i.e. is modelled only MBLA thermohydrodynamics. It is basic model for our researches.

The "humidity" model, i.e. model considers processes fog and cloud formation against the background of a thermohydrodynamics of "dry" MBLA. In this task are simulated:

Separately the taken cloud and fog;

The mode of simultaneous existence of a cloud and fog is revealed;

It is imitated daily continuous cloudiness;

The integrated vertical complex of fog and cloud is received - here we had to investigate in details a role of horizontal and vertical turbulence in formation of humidy processes;

"Theoretical" influence on radiation fog by means of thermal sources and upright the descending airflows is carried out; are defined time of a dispelling of fog; optimum regime of influence are picked logically up.

Influence of background wind on MPSA thermohydrodynamics is investigated - it can modelling creation and optimization of wind-shelter strips.

The Classical Three-Body Problem without Environment and with It. New Ideas and Approaches in the Theory of Dynamical Systems

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The three-body general problem as a problem of geodesic flows on the Riemannian manifold is formulated [1]. It is proved that a curved geometry with its local coordinate system allows us to detect new hidden symmetries of the internal motion of a dynamical system and reduce the three-body problem to the system of order 6th. It is shown that the equivalence between of the initial Newtonian three-body problem and developed representation provides coordinate transformations in combination with an underdefinished system of algebraic equations, that makes the system of geodesic equations *irreversible* relative to the evolution parameter, i.e. the length of the arc of the geodesic curve. The latter leads to branching of trajectories problem, which, depending on the geometry properties of the Riemannian space, can lead to chaos in the dynamical system. Equations of deviation of geodesic trajectories characterizing the behavior of the dynamic system as a function of the initial parameters of the problem are obtained. To describe the motion of a dynamical system influenced by external regular and stochastic forces, a system of stochastic equations (SDEs) is obtained. Using these SDEs, the partial differential equation of the second order is obtained for the joint probability distribution of the momentum and coordinate. A criterion for estimating the degree of deviation of probabilistic current tubes of geodesic trajectories in the phase and configuration spaces is formulated. The mathematical expectation of the transition probability between different asymptotic channels is determined taking into account the multichannel nature of the scattering.

Lastly, it is proved that the classical dynamical system, beginning with three bodies, has a so-called internal evolution time, which makes classical equations irreversible relative to this parameter. The latter leads to new ideas and possibilities for studying dynamical systems as mathematical as well as physical points of view.

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Diagram Using Some Practical Aspects

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From ancient times the practical work of the human being required the physical, geometric and other properties of different (but uniform) objects. In the primary school mathematics course to solve problems and draw up the tasks it takes great place for diagrams. The solution of the two-dimensional diagrams are based on the theory theorem, in particular, at any point on the rectangle diagonal, parallel lines of the rectangle sides, and in the inside of the given rectangle, the two rectangles are rectangular. Sometimes the simplest calculations can be made to answer the questions in the tasks that can be achieved using two-dimensional diagrams.

Problem of Statics of the Linear Thermoelasticity of the Microstretch Materials with Microtemperatures for a Half-Space

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We consider the statics case of the theory of linear thermoelasticity with microtemperatures and microstrech materials. The representation formula of differential equations obtained in the paper is expressed by means of four harmonic and four metaharmonic functions. These formulas are very convenient and useful in many particular problems for domains with concrete geometry. Here we demonstrate an application of these formulas to the III type boundary value problem for a half-space. Uniqueness theorems are proved. Solutions. are obtained in quadratures.

Boundary Value Problems of statics of Thermoelasticity of Bodies with Microstructure and Microtemperatures

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The paper deals with the equilibrium theory of thermoelasticity for elastic isotropic microstretch materials with microtempeatures and microdilatations. For the system of differential equations of equilibrium the fundamental matrix is constructed explicitly in terms of elementary functions. With the help of the corresponding Green identities the general integral representation formula of solutions by means of generalized layer and

Newtonian potentials are derived. The basic Dirichlet and Neumann type boundary value problems are formulated in appropriate function spaces and the uniqueness theorems are proved. The existence theorems for classical solutions are established by using the potential method.

Energy Control Issues

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We present our collaborative research with energy experts about some issues of energy control:

- Elaboration of the optimal parameters based on the Moran model for the HPP (Hydro Power plant) with reservoir under construction;
- Study of optimal regimes of work of HPP by means of the method of Dynamic Programming, reducing the constraints imposed in the previous works of the authors;
- Study of optimal regimes for the cascade systems;
- Control of multifunctional systems;
- Two models of optimal work of energy system (regional or for the single country) implemented by use of Dynamic or Mathematical Programming.

Based on the real, existing data we also present the results of practical character.

On the Theoretical and Applied Aspects of the Generating Functions' Method in Discrete Math Learning Courses

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The method of generating functions creates a powerful analytical apparatus of mathematical research. Its capabilities and theoretical and practical areas of use are diverse in the study of discrete objects. At the beginning of systematic teaching of this theory, methodically is preferable to set various simple and interesting practical motivational tasks and demonstrate an efficient use of the method of generating functions for the solution of the tasks.

The generating function $A(z)$ of the numerical sequence (a_n) is a power series $\sum\limits_{n\geq 0} a_n z^n$, where z can be considered as complex variable. Then obviously the question of convergence of the corresponding series arises.

$A(z)$ can also be considered as a formal power series and in such representation, if the problem of studying the asymptotic behavior of series' coefficients is excluded, the question of the series convergence can be removed. Then, even in the case of divergent series, we can formally conduct operations of differentiation and integration and these operations will be correct. In such cases, if for the sequence (a_n) given by recurrence we obtain close formed formula (i.e. a formula in which a_n can be represented as an explicit function of n), then these formulas considered as hypothetical can be proved by the mathematical induction.

By addition, multiplication, shift, transformation of variables. differentiation and integration the possibilities of using of these functions are widened.

In our talk we present different spheres of the use of generating functions and also present the methodical side of their teaching. In particular, certain algorithms of obtaining close formed formulas for sequences given by recurrence in various problems will be illustrated.

Periodic Field Configurations in a Theory of Scalar Fields and Phase Transitions

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The periodic field configurations have turned out to be a significant field configurations which interpolate between the stable vacuum configurations and unstable sphalerons sitting on the top of the potential barrier. In this process the transition from false vacuum to the true one takes place and at the finite energies below the potential barrier temperature assisted quantum tunneling is dominating whereas at higher energies (above the potential barrier) the process is pure classical (thermal activation).

The talk is devoted to a simple model in which periodic field configurations are found, indeed a scalar triplet with broken $SU(2)$ symmetry is considered. Classical equations are studied and a set of particular solutions obtained. The solutions are analyzed in view of phase transitions. Charged and neutral solutions are presented. These periodic field configurations are used in view of phase transitions between the thermally assisted quantum tunneling and pure classical process (thermal activation). The field configurations with zero and nonzero charges are analyzed and their contributions in phase transitions investigated.

Spectral Estimates for the Laplace and p-Laplace Neumann Operators in Space Domains

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In [1] we obtained lower estimates of $\mu_1(\Omega)$ in terms of a hyperbolic (conformal) radius of Ω for domains $\Omega \subset \mathbb{R}^2$ for a large class of non-convex domains. For space domains the theory of conformal mappings is not relevant.

We prove discreteness of the spectrum of the Neumann–Laplacian in a large class of non-convex space domains. The lower estimates of the first non-trivial eigenvalue are obtained in terms of geometric characteristics of homeomorphisms that induce composition operators on homogeneous Sobolev spaces $L^{1,p}$. The suggested method is based on Poincaré–Sobolev inequalities that are obtained with the help of the geometric theory of composition operators. A corresponding composition operators are induced by a generalizations of conformal homeomorphisms that are mappings of bounded 2-dilatation (weak 2-quasiconformal mappings).

The work is done jointly with *Alexander Ukhlov*.

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Boundary Value Problems for a Circular Ring with Triple-Porosity in the Case of an Elastic Cosserat Medium

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In the paper the two-dimensional version of the fully coupled linear theory of elasticity for solids with triple porosity is considered in the case of an elastic Cosserat medium [1]–[3]. The solutions are represented by means of three analytic functions of a complex variable and three solution of the Helmholtz equations [4]. The problems are solved when the components of the displacement vector or the stress tensor are known on the boundary of the concentric circular ring.

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Using a Small Parameter Method for Splitting of the Multi-Level Semi-Discrete Scheme for the Evolutionary Equation

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In the work, for evolutionary equations, multi-level (three and four-level) semi-discrete schemes are considered. With a small parameter method (with perturbation algorithm), these semi-discrete schemes are reduced to two-level schemes. Using the combination of the solutions of these schemes, an approximate solution to the original problem is constructed. In a Banach and a Hilbert spaces, estimate on the approximate solution error are proved using properties of semigroups and associated polynomials under minimal assumptions about the smoothness of the data of the problem.

Perturbation Algorithm for Numerical Realization of Difference Scheme of Parabolic Equation

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In the work, using the perturbation algorithm, computer realization of a purely implicit three-level difference scheme for spatial single-dimensional evolutionary problem has been implemented. In MatLab the relevant program and user interface has created. Numerical calculations are executed and the corresponding graphs are presented.

The interface allows users to perform numerical realization of this type of evolutionary problems without knowing the perturbation algorithm. In addition to the specificity of the algorithm, the interface is built in such a way that it allows the customer to gradually solve the initial problem, which in turn will useful to carry out a number of research numerical experiments.

An Overwiev Intuitionistic Fuzzy Soft Supratopological Spaces

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In this paper, we introduce intuitionistic fuzzy soft supratopological spaceand then we define the notions of intuitionistic fuzzy soft supra closure, intuitionistic fuzzy soft supra interior. Later we investigate some of their important properties. We also consider the notions of fuzzy strongly soft supra connected, fuzzy soft supra continuous mapping and intuitionistic fuzzy soft supra compact. We investigate many properties relations and characterizations of this notions.

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Separation Axioms in Supra Soft Bitopological Spaces

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The concept of soft sets was introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain objects. Later topological stuructures of soft sets have been studied by some authors in recent years. M. Shabir and M. Naz presented soft topological spaces and studied some important properties of them. Many researcher studied some of basic concepts and properties of soft topological spaces. S. A. El-Sheikh and A. M. Abd. El-latif introduced the concept of supra soft topological spaces. Supra soft topological spaces are generalization of soft topological spaces. Ittanagi defined the notion of soft bitopological space which is given over an initial universal set X with fixed set of parameters. The purpose of this paper is to introduce the notions of some separation axioms in supra soft bitopological spaces and investigate some of their properties.

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On the Integral Invariants of Line Geometry

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As known the geometry of a trajectory surfaces tracing by an oriented line (spear) is important in line geometry and spatial kinematics. Until early 1980s,although two real integral invariants, the pitch of angle λ_x and the pitch ℓ_x of an x-trajectory surface were known, any dual invariant of the surface were not. Because of the deficiency, line geometry wasn't being sufficiently studied by using dual quantities.

A global dual invariant, Λ_x, of an x-closed trajectory surface is introduced and shown that there is a magic relation between the real invariants, $\Lambda_x = \lambda_x - \varepsilon\ell_x$, [1]. It gives suitable relations, such as $\Lambda_x = 2\pi - A_x = \oint G_x\, ds$ or $\lambda_x = 2\pi - a_x = \oint g_x\, ds$ and $\ell_x = a_x^* = \oiint (\partial_u + \partial_v)\, du\, dv$ which have the new geometric interpretations of x-trajectory surface where a_x is the measure of the spherical area on the unit sphere described by the generator of x-closed trajectory surface and ∂_u and ∂_v are the distribution parameters of the principal surfaces of the $X(u;v)$-closed congruence. Therefore all the relations between the global invariants λ_x, ℓ_x, a_x, a_x^*, g_x, g_x^*, K, T, σ and s_1 of x-c.t.s. are worth reconsidering in view of the new geometric explanations. Thus, some new results and new explanations are gained [2], [3]. Furthermore, as a limit position of the surface, some new theorems and comments related to space curves are obtained.

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Bounded Operators on Weighted Besov Spaces of Holomorphic Functions on Polydiscs

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Assuming that S is the space of functions of regular variation (see [2]) and $\omega = (\omega_1, \ldots, \omega_n)$, $\omega_j \in S$, by $B_p(\omega)$ we denote the class of all holomorphic functions defined on the polydisk U^n such that

$$\|f\|^p_{B_p(\omega)} = \int\limits_{U^n} |Df(z)|^p \prod_{j=1}^{n} \frac{\omega_j(1-|z_j|)\, dm_{2n}(z)}{(1-|z_j|^2)^{2-p}} < +\infty,$$

where $dm_{2n}(z)$ is the $2n$-dimensional Lebesgue measure on U^n and D stands for a special fractional derivative of f defined here. For properties of holomorphic Besov spaces see [1].

We consider the boundedness of little Hankel operator $h^\alpha_g(f)$ in $B_p(\omega)$. For the case $0 < p < 1$ and for the case $p = 1$ we have the following results

Theorem 1. *Let $0 < p \le 1$, $f \in B_p(\omega)$ (or $f \in \overline{B}^p(\omega)$), $g \in L^\infty(U^n)$. Then $h^\alpha_g(f) \in \overline{B}_p(\omega)$ if and only if $\alpha_j > \alpha_{\omega_j}/p - 2$, $1 \le j \le n$.*

The case $p > 1$ is different from the cases of $0 < p < 1$ and from the case of $p = 1$. Here we have the following

Theorem 2. *Let $1 < p < +\infty$, $f \in B_p(\omega)$ (or $f \in \overline{B}_p(\omega)$), $g \in L^\infty(U^n)$. Then if $\alpha_j > \alpha_{\omega_j}$, $1 \le j \le n$ then $h^\alpha_g(f) \in \overline{B}_p(\omega)$.*

In this paper we consider also the generalized Berezin type operators $B^\alpha_g(f)$ on $B_p(\omega)$ (and on $L_p(\omega)$) and prove some theorems about the boundedness of these operators.

We have the following results:

1. for the case of $0 < p < 1$ we have

Theorem 3. *Let $0 < p < +\infty$, $p \not{1}$, $f \in B_p(\omega)$ (or $f \in \overline{B}_p(\omega)$), $g \in L^\infty(U^n)$ and let $\alpha_j > \alpha_{\omega_j}/p - 2$, $1 \le j \le n$. Then $B^\alpha_g(f) \in L^p(\omega)$.*

2. we consider now the case of $p = 1$.

Theorem 4. *Let $f \in B_1(\omega)$ (or $f \in \overline{B}_1(\omega)$), $g \in L^\infty(U^n)$. Then $B_g^\alpha(f) \in L_1(\omega)$ if and only if $\alpha_j > \alpha_{\omega_j}$, $1 \le j \le n$.*

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On a Boundary Value Problem with Infinite Index

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In the work it is investigated Riemann boundary value problem in a unit circle $D^+ = \{z; |z| < 1\}$ with the following setting:

Problem R

Determine analytic in $D^+ \cup D^-$ function $\varphi(z)$, $\varphi(\infty) = 0$ such that the following holds:

$$\lim_{r \to 1-0} \big\| \varphi^+(rt) - a(t)\varphi^-(r^{-1}t) - f(t) \big\|_{L^p(\rho)} = 0, \tag{1}$$

where $1 \le p < \infty$, $\rho(t) = \prod\limits_{k=1}^{\infty} |t_k - t|^{\delta_k}$, $\delta_k > 0$, $\sum\limits_{k=1}^{\infty} \delta_k < \infty$.

On a solution

In the case $1 < p < \infty$ it is shown that problem (1) is normally solvable. In other words, the homogeneous problem has a finite number of linearly independent solutions, and the inhomogeneous problem is solvable for any function $f \in L^p(\rho)$. If $p = 1$, then the general solution of the homogeneous problem (1) can be represented in the form:

$$\varphi_0(z) = \sum_{k=1}^{\infty} \frac{A_k}{t_k - z}\,,$$

where $\sum\limits_{k=1}^{\infty} A_k < \infty$. Thus, the general solution of the problem (1) can be represented in the form: $\varphi(z) = \varphi_0(z) + \varphi_1(z)$, where

$$\varphi_1(z) = \sum_{k=1}^{\infty} \varphi_{1k}(z)$$

and

$$\varphi_{1k}(z) = \frac{1}{2\pi i(t_k - z)} \int\limits_{T_k} \frac{f(t)(t_k - t)}{t - z}\, dt.$$

Besides, $T = \cup T_k$, where T_k, $k = 1, 2, \dots$, intervals are mutually disjoint.

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სოფლის მეურნეობაში საინფორმაციო ტექნოლოგიების გამოყენების აუცილებლობის შესახებ

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ბოლო 10-15 წელია მსოფლიოს განვითარებული ქვეყნები დიდი წარმატებით იყენებენ საინფორმაციო ტექნოლოგიებს სოფლის მეურნეობის წინაშე მდგარი ამოცანების გადასაწყვეტად, ხოლო საქართველო მიეკუთვნება იმ განვითარებად ქვეყნებს, სადაც, ამ მხრივ, არსებობს დიდი ჩამორჩენა. ევროკავშირის ქვეყნების, აშშ, კანადისა და ზოგიერთი სხვა ქვეყნის გამოცდილების შესწავლისა და ანალიზის შედეგად შეიძლება დასკვნის გაკეთება, რომ სასოფლო-სამეურნეო წარმოების ინტენსიური განვითარება, მაღალი ეფექტურობა და ეკოლოგიური ხარისხიანი სასოფლო პროდუქტების მიღება წარმოუდგენელია თანამედროვე საინფორმაციო ტექნოლოგიების გარეშე. მოხსენებაში ნაჩვენებია ქვეყნის სოფლის მეურნეობის რაიონული და რეგიონალური (სამხარეო) საინფორმაციო სისტემების შექმნის აუცილებლობა. მიმდინარეობს მუშაობა სისტემების ზოგად ამოცანათა ჩამონათვალის განსაზღვრისა და მათი ფორმალიზებული აღწერისთვის.

სამთო მომპოვებელი და გამამდიდრებელი საწარმოების სამრეწველო ტრანსპორტის მართვის ავტომატიზებული სისტემის სტრუქტურა

ნუგზარ იაშვილი

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სამთო-მომპოვებელი და გამამდიდრებელი საწარმოების ადგილმდებარეობა მნიშვნელოვნად განსაზღვრავს შიდა სატრანსპორტო გადაზიდვებისათვის სამრეწველო ტრანსპორტის სხვადასხვა სახეობის გამოყენებას.

სამრეწველო ტრანსპორტის ტრადიციული სახეების გარდა, სამთო საწარმოებში გამოყენებას ჰპოვებს ტრანსპორტის ისეთი სახეობები, როგორებიცაა კიდული საბაგირო გზები, ჰიდრავლიკურ და პნევმატიკური მილსადენები, კონვეირული და სხვა.

ზოგადად სამრეწველო ტრანსპორტს ახასიათებს სტაციონარობა, ტვირთების შედარებით ვიწრო სპეციალიზაცია და მათი ნაკადების ცალმხრივი მიმართულება.

ამიტომ სამთო-მომპოვებელი და გამამდიდრებელი საწარმოებში მიზანშეწონილია სამრეწველო ტრანსპორტის სხვადასხვა სახეობის კომპლექსური გამოყენება. ეს კი თავის მხრივ განსაზღვრავს გადასაწყვეტ ამოცანებს.

აქ უნდა ავღნიშნოთ ის ფაქტი, რომ თუ ტრადიციული, საავტომობილო და სარკინიგზო ტრანსპორტის ამოცანები, მათი რეალიზაციის ალგორითმები და პროგრამები ცნობილია და დანერგილია მთელ რიგ საწარმოში, ტრანსპორტის სპეციალურ სახეობათა მიმართ არ არსებობს არა თუ დამუშავებული ალგორითმები და პროგრამები, არამედ მსგავსი საკითხები განხილულიც კი არ არის.

სამთო-მომპოვებელი და გამამდიდრებელი საწარმოების ტრანსპორტის ოპერატიული მართვის სისტემაში გადასაწყვეტი ამოცანები შეიძლება დაჯგუფდეს სამ ჯგუფად:

1. საავტომობილო ტრანსპორტის ამოცანები;
2. სარკინიგზო ტრანსპორტის ამოცანები;
3. ტრანსპორტის სპეციალური სახეობების ამოცანები.

თითოეული აღნიშნული ჯგუფისთვის განსაზღვრულია ამოცანათა შემდეგი კომპლექსების გადაწყვეტა: ტრანსპორტის ერთეულების მუშაობის დაგეგმვა, დღიური და საკვირაო გრაფიკების გათვლები და შედგენა; ტრანსპორტის სახეობათა გამოყენების არრიხცვა და ანალიზი; ოპერატიული ანგარიშების შედგენა და სამრეწველო ტრანსპორტის ცალკეულ სახეობათა ოპერატიულ-დისპეტჩერული მართვა. აქვე შესაძლებელია ტრანსპორტის ცალკეულ სახეობათა ერთობლივი მუშაობის ორგანიზება (თუ ასეთი კონკრეტულ საწარმოში არსებობს).

ტრანსპორტის მართვის ავტომატიზებულ სისტემაში განსაზღვრულია სამთო-მომპოვებელი და გამამდიდრებელი საწარმოებისათვის კონკრეტული ამოცანების რეალიზაცია.

მიკროპროცესორული გამზომი ხელსაწყოს ფუნქციონირების მათემატიკური მოდელი

ნუგზარ იაშვილი, გიორგი იაშვილი

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მიუხედავად იმისა, რომ გამზომი ხელსაწყოები მათი ფუნქციური დანიშნულებისა და აგების სტრუქტურით მნიშვნელოვნად განსხვავდებიან ერთმანეთისაგან, ალბათ მაინც შესაძლებელია მათი წარმოდგენა ზოგადი ბლოკის სქემის სახით. ეს განპირობებულია იმითაც, რომ ნებისმიერ გამზომ ხელსაწყოში შესაძლებელია მისი მთავარი, განმსაზღვრელი კვანძებისა და ბლოკების გამოყოფა. ეს, თავის მხრივ, საშუალებას გვაძლევს მოვახდინოთ გამზომი ხელსაწყოს ფორმალიზებული (მათემატიკური) აღწერა და მისი ფუნქციონირების მოდელის აგება.

დღევანდელი ნებისმიერი გამზომი ხელსაწყოს აუცილებელი ნაწილია მიკროპროცესორული ბლოკი, რომლის დანიშნულებაა ხელსაწყოს სენსორიდან მიღებული სიგნალის მიღება, გარდაქმნა, დამუშავება და გაზომვის შედეგის ინდიკაციის ტაბლოზე გამოსახვა და მართვის სიგნალების გამომუშავება შემსრულებელი მოწყობილობებისათვის.

მიკროელექტრონიკის სწრაფმა განვითარებამ განაპირობა ე. წ. „ჭკვიანი“ (ინტელექტუალური) გამზომი ხელსაწყოების გაჩენა, რომელთაც საფუძვლად უდევს ინფორმაციის პროგრამული მართვა დისკრეტულ (ციფრულ) ფორმაში. ამ ახალი მოწყობილობის მთავარი შემადგენელია მიკროპროცესორები, რომელთა პროექტირებისა და დამუშავებისათვის აუცილებელია მათემატიკური მოდელირების მეთოდების გამოყენება.

ჩვეულებრივ, გაზომვის პროცესების მათემატიკურ მოდელებში აისახება გასაზომი ობიექტის, მოვლენის თუ პროცესის ყველაზე მთავარი და მნიშვნელოვანი თვისებები და მახასიათებელი ნიშნები. მათემატიკური მოდელირების შედეგად მიიღება ახალი ინფორმაცია, რომელიც საშუალებას იძლევა უფრო ღრმად და ზუსტად აღვწეროთ გასაზომი ობიექტი, რათა ახალი მიკროპროცესორული მოწყობილობისა და სისტემის დამუშავების პროცესში გათვალისწინებული იქნას ყველა აუცილებელი პირობები და მოთხოვნები. მოხსენებაში განხილულია გამზომი ხელსაწყოს კონკრეტული მაგალითი; ბუნებრივი აირის გაჟონვის აღმოსაჩენი მოწყობილობის ფუნქციონირების მოდელის აგება ობიექტის დამახასიათებელი ნიშან-თვისებებისა და მიკროპროცესორული ბლოკის პარამეტრების გათვალისწინებით.

პიროვნების იდენტიფიცირებისათვის ბიომეტრიული მახასიათებლების გამოყენება

ლევან იმნაიშვილი, გიორგი იაშვილი, მაგული ბედინეიშვილი

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პიროვნების დაშვების კონტროლის ერთ-ერთ პერსპექტიულ სახეს წარმოადგენს დაშვების სისტემებში ბიომეტრიული მეთოდების გამოყენება, რომელიც განსაკუთრებით ფართოდ გავრცელდა ბოლო წლებში.

პიროვნების დაშვების კონტროლის ტრადიციული სისტემებისაგან განსხვავებით დაშვების ბიომეტრიულ სისტემებში იდენტიფიკატორებად გამოიყენება პიროვნების ბიომეტრიული მახასიათებლები: თითი, სახე, ხელი, თვალის გარსი და სხვა.

დაშვების სისტემებში ბიომეტრიული მახასიათებლების გამოყენება საშუალებას იძლევა:

- ავლკვეთოთ დაცულ ტერიტორიასა და შენობებში არასანქცირებული დაშვება;
- უზრუნველყოს საპასუხისმგებლო ობიექტებზე მხოლოდ სერტიფიცირებული სპეციალისტების დაშვება;
- გამოირიცხოს უხერხულობები რომლებიც დაკავშირებულია გასაღებების, ბარათების, ქეტონების და პაროლების დაკარგვასა და დაზიანებებთან;
- უზრუნველყოს შენობებსა და ტერიტორიებზე დაშვებულ პირთა აღრიცხვა;

განსაკუთრებული და სპეციალური მნიშვნელობის მქონე დაწესებულებებისათვის რეკომენდირებულია დაშვების ბიომეტრიულ სისტემაში რამდენიმე ბიომეტრიული მახასიათებლის გამოყენება (ე. წ. „მულტი ბიომეტრიული სისტემა“), რომელიც მაღალი ეფექტურობით იძლევა პიროვნების ამოცნობის იდენტიფიცირების საშუალებას ანუ მისი ამოცნობის საშუალებას ერთდროულად სხვადასხვა ბიომეტრიული პარამეტრებით. მაგალითად, თითის მოხაზულობით და სახის მოყვანილობით ან თითის მოხაზულობით და თვალის გარსით.

PDE Based Method for Image Enhancing and Image Restoration

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In this work we present an new filters for image enhancing and image restoration processes. We use the proposed filter that is completely adaptive and it changes proportional to structure of the image. This model has some advantages such as deblurring and denoising and fully adaptivity. It can be used practically in enhancing processes. We use different test images to show the validity of this method. Numerical experiments illustrate the efficiency of the proposed method.

Keywords: Perona–Malik filter, shock filter, deblurring, PDE based image restoration, fractional differential equation.

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On the Yokoi's Invariant Value of Certain Real Quadratic Fields with the Period Eight

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Let $K = Q(\sqrt{d})$ be a real quadratic field, where d is a positive square-free integer congruent to 1 modulo 4. In this paper, we obtain some conditions for Yokoi's d-invariant value to be zero with period $k_d = 8$ using the explicit form of the fundamental unit.

2010 AMS Subject Classification: Primary 11A55; Secondary 11R27.

Key Words: Continued fraction, fundamental unit.

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The Boundary-Transmission Problem of Statics

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The present thesis deals with the problems of Mathematical Physics, in particular three-dimensional boundary-transmission problems of statics of thermo-elasticity theory for hemitropic solids.

One of the most essential results is investigation of the uniqueness questions. The case is that solutions to the problem should be sought in the space of vector functions which are only bounded at infinity.

The existence of solutions is proved by the potential method and the theory of singular integral equations.

A Problem of Plane Asymmetric Elasticity for a Perforated Rectangular Domain

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In this paper the plane boundary value problem of the asymmetric theory of elasticity for a perforated domain is considered. The domain is a square with circular holes located in a certain way. The problem of stretch-press of such a plate is solved approximately [1].

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The Beta Function, New Properties and Applications

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We have defined the Euler Beta function of imaginary parameters [1] and have shown that

$$B(ix,-ix)=\int\limits_0^1 dt\, t^{ix-1}(1-t)^{-ix-1}=\int\limits_0^\infty dt\, t^{ix-1}=2\pi\delta(x). \tag{1}$$

From the equalities (1), one gets:

$$\int\limits_{-\infty}^{+\infty}\frac{du}{1+\exp(u)}\,\cos(xu)=\int\limits_{-\infty}^{+\infty}du\,\frac{\exp(u)}{1+\exp(u)}\,\cos(xu)=\pi\delta(x). \tag{2}$$

Differentiating the expression (1) one can write (see also [2]):

$$\int\limits_0^1 dt\, t^{ix-1}(1-t)^{-ix-1}\ln[t/(1-t)]=\int\limits_0^1 dt\, t^{ix-1}\ln(t)=-2\pi i\delta'(x).$$

Due to the formulas (2), one obtains:

$$\int\limits_{-\infty}^{+\infty}\frac{du}{1+\exp(u)}\,\sin(xu)u=\int\limits_{-\infty}^{+\infty}du\,\frac{\exp(u)}{1+\exp(u)}\,\sin(xu)u=-\pi\delta'(x).$$

The method developed in [1] is connected with many new results, some of them useful in avoiding the quantum mechanical difficulties.

Acknowledgement. This work was supported by Georgian Shota Rustaveli National Science Foundation (grant FR/ 417/6-100/14).

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The Special Functions, the New Relations

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Using the next integral formula (see, e.g., [1, p. 960])

$$Q_\nu^\mu(z) = \exp(i\pi\mu)\,\frac{\Gamma(\nu+1)}{\Gamma(\nu-\mu+1)}\int\limits_0^\infty dt\,\cosh(\mu t)\big(z+\sqrt{z^2-1}\cosh(t)\big)^{-\nu-1},$$

$$\operatorname{Re}(\nu+\mu) > -1,\ \ \nu \neq -1,-2,-3,\dots,|\arg(z\pm 1)| < \pi,$$

one can show that the relation holds [2]:

$$Q_\nu^{i\tau}(z) = \exp(-\pi\tau)\,\frac{\Gamma(\nu+i\tau+1)}{\Gamma(\nu+1)}\,Q_\nu(z), \tag{1}$$

$$\operatorname{Im}(\nu) = 0,\ \ \operatorname{Re}(\nu) > -1,\ \ |\arg(z-1)| < \pi,$$

where $Q_\nu^\mu(z)$ and $Q_\nu(z)$ are the well-known Legendre functions of the second kind and $\Gamma(z)$ is the Euler gamma function.

Besides, due to the asymptotic formula (see, e.g., [3, p. 220]):

$$Q_\nu(z) = -\frac{\ln(z-1)}{2\Gamma(\nu+1)},\ \ z\to 1,\ \ \nu \neq -1,-2,-3,\dots,|\arg(z-1)| < \pi,$$

from the equality (1) one obtains:

$$Q_\nu^{i\tau}(z) = -\frac{1}{2}\,\exp(-\pi\tau)\,\frac{\Gamma(\nu+i\tau+1)}{\Gamma^2(\nu+1)}\,\ln(z-1),\ \ z\to 1, \tag{2}$$

$$\operatorname{Im}\nu = 0,\ \ \operatorname{Re}\nu > -1,\ \ |\arg(z-1)| < \pi.$$

The results obtained are useful in avoiding the quantum mechanical problems. Using the formulas (1) and (2) we have derived the transition amplitude of two charged particles of continuous spectrum in the light photons approximation.

Acknowledgement. This work was supported by Georgian Shota Rustaveli National Science Foundation (Grant FR/417/6-100/14).

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About One Method of Solution of Elliptic Kirchhoff Type Equation

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Let us consider the following boundary value problem

$$-\varphi\bigg(\int\limits_{\Omega}|\nabla w|^2\,dx\bigg)\Delta w=f(x),\quad x\in\Omega, \tag{1}$$

$$w(x)=0,\quad x\in\partial\Omega, \tag{2}$$

where Ω is an open subset of $\mathbb{R}^n$, $n\geq 1$, and $\partial\Omega$ is its boundary. The function $f(x)$ is twice continuously differentiable function on Ω and function $\varphi(z)$, $0\leq z<\infty$, satisfies the condition

$$\varphi(z)\geq\alpha>0.$$

In [1] the problem (1), (2) is studied when $n=1$. For the solution is used Chipot's approach and accuracy of the method is discussed. Here also are given numerical examples. In [2] the problem (1), (2) is studied when $n=2$. For the solution is used Chipot's approach and accuracy of the method is discussed.

In this paper, we use Chipot's approach to solve the problem (1), (2). For this purpose we develop the computer program in Matlab. Furthermore, we give the numerical example for $n=3$.

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Line and Grid Composed Methods

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Unlike line and grid classical methods to solve partial differential equations line and grid methods based on the high accuracy Sh. Mikeladze formula (so-called method without saturation), which approximation and convergence order depends on the number of knots (lines) are considered. The composed methods give possibility to use discretization of the received (in the result of the first discretization) subintervals the second time. This discretization process can be repeated so many times till the necessary number of simultaneous algebraic or ordinary differential equations is received. In the case of the composed method of lines first discretization by m inner knots gives m simultaneous ordinary differential equations. In the case of the composed grid method (first discretization according both variables) gives mn (m^2) simultaneous algebraic equations. In the result of using composed method several times, the number of the equations and variables increases in a square of times of each use. Usage of the composed method influences the order of approximation and convergence.

On One Family of Separable Primary Groups

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The family of separable primary groups, where additive group of endomorphisms ring of the group is a direct sum of additive group of separable closed unital subring and a group of small endomorphisms, is considered. Relying on the results of A. Korner and A. Moskalenko it is shown, that the cotorsion hulls of these groups are not fully transitive.

On the Cotorsion Hulls in the Class of Primary Groups

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It is shown that there is the class of separable primary groups where the basic subgroup of group is unbounded, is not more than least uncountable cardinality but the class consists with nonisomorphic groups which cardinality is more than the least uncountable cardinal and the cotorsion hull of each group is not fully transitive.

Integration Mathematical and Computer Models of the Information Warfare

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New mathematical and computer models of the Information Warfare are being considered. An attempt has been made to unify the mathematical model of attracting adepts given in the monograph of A. A. Samarskii, A. P. Mikhailov [1] with the mathematical and computer model of flows outlined in the works T. I. Chilachava, N. G. Kereselidze [2]–[4].

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Quantum-Chemical Study of the Propensity of the Amino Acid Pairs for the Peptide Bond Formation

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The Modern method of quantum chemistry – Density Functional Theory (DFT) is used for the quantitative description of the formation of peptide bond between amino acid pairs. The formula of propensity of amino acids (Kp) for the peptide bond formation is constructed, which in turn is a function of six variables: both length and order of the CO and NH bonds, (R_{CO}, R_{NH}, P_{CO}, P_{NH}), the activation energy for the formation of the peptide bond ($\Delta E^{\#}$), and difference between charges of the carbon atom of the carbonyl group and the amino nitrogen atom (Δq). By means of the proposed formula the Kp parameter for 400 amino acid pairs was calculated. Among them only 26 amino acid pairs are most likely to take part in the synthesis of proteins that have been selected based on the value of the parameter Kp. This approach may have important meaning for quantitative description of the amino acid sequences in proteins.

On Hilbert H^*-Bimodules

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In this paper, we introduce the notion of Imprimitivity Hilbert H^*-bimodule and describe some properties of it. Moreover, we show that if $\mathcal{A}$ and $\mathcal{B}$ are proper and commutative H^*-algebras, ${}_{\mathcal{A}}E_{\mathcal{B}}$ is a Hilbert H^*-bimodule and e_1 is a minimal projection in $\mathcal{A}$ with ${}_{\mathcal{A}}[x|x] = e_1$ for some $x \in \mathcal{A}$, then $[x|x]_{\mathcal{B}}$ is a minimal projection in $\mathcal{B}$, too. Furthermore, the existence of orthonormal bases for such spaces are studied.

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Some Aspects of Using the Internet in the Process of Learning Mathematics

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It's no secret that many students have problems with mastering mathematical subjects. To advance their knowledge in mathematics, it is very important not only to keep in mind an appropriate theoretical material, but also to learn how to solve problems independently. By solving mathematical problems of varying degrees of complexity, students will be able to gain a useful experience, as well as to assess the level of their knowledge. Actually, this is the best way to prepare for colloquiums, exams or math tests.

Nowadays, the Internet can substantially help students in this respect, because there are special sites for self-studying mathematics. These sites suggest different options. As a rule, on these sites students can find:

- various problems, solving which they can improve their mathematical skills;
- online calculators by using which students will be able to enter the task data and get an answer in the form of a detailed solution;
- theoretical material concerning various mathematical themes with relevant examples;
- manual material including different tables and lists of formulas that are often used in the process of solving mathematical problems.

Therefore, for students who study mathematics courses, it is necessary to provide them with information about the most well-known Internet resources of mathematical orientation:

- sites containing theoretical information (e.g., in the form of lecture courses);
- sites containing various mathematical packages and information about their usage;
- sites that contain examples, tasks and their solutions.

Undoubtedly, the Internet is an underutilized resource, but the academic personal of universities needs to better study these opportunities in order to use them for improving students knowledge of mathematics.

Summarizing all the said above, the Internet becomes a significant resource in mathematics education. The diversity and interactive nature of the materials presented by Internet sites enable students to learn and visualize mathematical notions with the aid of a new methodology.

Interaction of the Zonal Flows with the Dift Waves in the Ionosphere

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Influence of the large-scale zonal flows and magnetic fields on the relative short-scale ULF electromagnetic waves in the dissipative ionosphere in the presence of a smooth inhomogeneous zonal wind (shear flow) is studied. A broad spectrum of Alfvenic-like electromagnetic fluctuations appears from electromagnetic drift turbulence and evidence of the existence of magnetic fluctuations in the shear flow region is shown in the experiments. In present work one possible theoretical explanation of the generation of electromagnetic fluctuations in DW-ZF systems is given. We show that the transient growth substantially exceeds the growth of the classical dissipative trapped-particle instability of the system. Excitation of electromagnetic fluctuations in such systems leads to the Attenuation-suppression of the short-scale turbulence. Also the numerical treatment of the satellite data is carried out. Influence of Bursty Bulk Flow (BBF) on the ionospheric plasma is investigated. It is shown, that it causes essential thinning of the plasma sheet at developing stage during passage inside it, but at the recovery stage plasma sheet thickening occurs and its size exceeds much the initial one.

About Some Application of Data Mining for Managements

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In the report I will tell how to use real-statistic program for excel during teaching research methods for managements.

Stability of Thin-Walled Spatial Systems with Discontinuous Parameters

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It is developed t3eh method of analysis of having holes and cuts shells and plates in conditions of linear and non-linear deformation that gives the possibility to determine with same precision the stresses and moments in continual area as well as in adjacent of edges and vertexes.

The obtained for analysis of having ribs and breaks shells are design formulae gives the possibility to describe all singularities of variable of the components of mode of deformation in adjacent of violation of regularity, express changes and distribution of stresses and moments in process of loading.

It is studied the simplified variant of solution, methodology for reducing of having equivalent bending stiffness single-layered plate that leads to essential simplification of calculations without significant losses in precision. Firstly are solved new problems of analysis of having cuts and holes sandwich plates in conditions of different supporting on edges and various specific loadings.

ლებეგის ინტეგრალის გამოთვლა უნიმორფულად განაწილებული მიმდევრობების საშუალებით

თენგიზ ქირია

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კოლმოგოროვის გაძლიერებული დიდ რიცხვთა კანონის გარკვეული მოდიფიცირების გამოიყენებით შესაძლებელი გახდა ბახას და შოისენგერის (2002) შედეგების გაფართობა მაქსიმალური რაოდენობა თანაბრადგანაწილებული მიმდევრობებისთვის $(0,1)$-ზე, რომელიც მკაცრად მოიცავს ყველა $(\{\alpha_n\})_{n\in\mathbb{N}}$ სახის მიმდევრობებს α ირაციონალობით და გაჩნია სრული ზომა ℓ_1^∞, სადაც ℓ_1^∞ არის წრფივი ლებეგის ზომის უსასრულო ხარისხი $(0,1)$-ზე.

Amply Cofinitely e-Supplemented Modules

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In this work, amply cofinitely e-supplemented modules are defined and some properties of these modules are investigated. It is proved that every factor module and every homomorphic image of an amply cofinitely e-supplemented module are amply cofinitely e-supplemented.

Results

Definition 1. Let M be an R-module. If every cofinite essential submodule of M has ample supplements in M, then M is called an amply cofinitely e-supplemented module.

Lemma 2. *If M is a π-projective and cofinitely e-supplemented module, then M is an amply cofinitely e-supplemented module.*

Corollary 3. *If M is a π-projective and cofinitely e-supplemented module, then M is an amply cofinitely e-supplemented module.*

Lemma 4. *Let R be any ring. Then every R-module is cofinitely e-supplemented if and only if every R-module is amply cofinitely e-supplemented.*

Key words: Cofinite submodules, essential submodules, small submodules, supplemented modules.

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An Application of Ontology Modeling in Semantic Web

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From its beginning, development of semantic web technologies was closely related to the Internet. The name itself, Semantic Web, was introduced by Tim Berners-Lee, who was a founder of this scientific direction. The main idea of the semantic web is to have knowledge available for wide auditory (the purpose of WWW itself) and to utilize this knowledge by developing systems for searching, browsing and evaluation. Thus, main technologies in semantic web are knowledge representation formats and different forms of knowledge.

Semantic Web is a collection of different technologies, where most of them is already standardized. The main purpose of these technologies is to describe semantic content of the web, i.e. their meaning and sense, in the format understood by computers. Such descriptions are called ontologies and the languages, on which ontologies are written – the ontology languages. Nowadays, the main research is concentrated on the ontology and logic layers.

The purpose of this talk is to present the author's research project which is going to be implemented together with her high-school students. The aim of the project is to build and implement an ontology model of Georgian touristic places. The resulting ontology potentially can be integrated in the semantic search engines.

Regional Climate Prediction System for South Caucasus Region

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To assess the socio-economic effects caused by climate change currently and in future individual countries need climate change scenarios provided by mathematical modelling of the climate system according to different socioeconomic scenarios defining the conditions of development of the society in the future. For South Caucasus region Regional Climate Model (RegCM4) was used. A model was run on the GRENA's cluster. Simulation was done (N 40°30′ − 47°; W 39°25′ − 44°) with maximal horizontal resolution of 20km, admissible in the area. Initial and boundary conditions were taken from EH5OM (MPI, Hamburg), global model output data (existing from 1941–2100) and A1B socio-economic scenarios. The simulation was done to cover 1959 to 2100 inclusively. Before assessing future trend of climate parameters model was bias corrected and uncertainties evaluated. Besides of changes in multiyear mean values of main climate parameters likely changes in extremes also have been analyzed. This extreme indices such as the number of hot and frost days, number of heavy rains, duration of heating and cooling periods also vegetation periods and etc. are very important for agriculture, energy, health and almost all sectors.

Acknowledgment. The research leading to these results has been co-funded by the European Commission under the H2020 Research Infrastructures contract no. 675121 (project VI-SEEM).

განრიგთა თეორიის კერძო ამოცანა შეზღუდული დამატებითი რესურსების პირობებში

ქეთევან კუთხაშვილი

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განიხილება განრიგთა თეორიის ამოცანა: საჭიროა შესრულდეს დავალებათა გარკვეული სიმრავლე რესურსების გარკვეული რაოდენობის და მომსახურების საშუალებებით. ჩვენი მიზანია აიგოს დავალებათა შესრულების თანმიმდევრობის ოტიმალური განრიგი ისე, რომ დაცული იყოს გარკვეული პირობები. ზოგადად ასეთი ამოცანა არაპოლინომიალური სირთულისაა და მხოლოდ კონკრეტულ შემთხვევებში ხერხდება პოლინომიალური სირთულის ალგორითმის აგება.

სამუშაო ეძღვნება პოლინომიალური ალგორითმის აგებას და მისი ეფექტურობის დადგენას ისეთი კონკრეტული ამოცანისათვის, როდესაც დავალებათა შესრულება ხდება ერთსაფეხურა პროცესორების საშუალებით, პროცესორების სისტემა ნაწილობრივ ურთიერთშეცვლადია, არც ნაწილობითი დალაგების სიმრავლე და არც დამატებითი რესურსების სიმრავლე არ არის ცარიელი. თითოეული დავალების შესრულებას სხვადასხვა დამატებითი რესურსის პირობებში სხვადასხვა საფასური შეესაბამება, ხოლო თითოეული დავალების შესრულება ნებისმიერ პროცესორზე მუდმივი სიდიდეა. ოპტიმალობის ზომად აღებულია

განრიგის სიგრძე დროის თვალსაზრისით და სისტემის მაქსიმალური ღირებულება. დადგენილია ალგორითმის ეფექტურობა.

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A Note on Relationships between Moments

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Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space, $\xi : \Omega \to \mathbb{R}^1$ be a random variable and $\mathbb{E}$ be the mathematical expectation symbol.

Theorem. *Let $p > q > 0$ be any numbers and for some constant $C \geq 1$ we have*

$$\big\{\mathbb{E}|\xi(\omega)|^p\,\mathbb{P}(d\omega)\big\}^{1/p} \leq C\big\{\mathbb{E}|\xi(\omega)|^q\,\mathbb{P}(d\omega)\big\}^{1/q}.$$

Then for any r, s, $0 < r, s \leq p$, the following inequality is fulfilled

$$\big\{\mathbb{E}|\xi(\omega)|^r\,\mathbb{P}(d\omega)\big\}^{1/r} \leq C^\beta\big\{\mathbb{E}|\xi(\omega)|^s\,\mathbb{P}(d\omega)\big\}^{1/s},$$

where

$$\beta = \begin{cases} 0, & \text{if } 0 < r \leq s \leq p, \\ 1, & \text{if } q \leq s < r \leq p, \\ \dfrac{q(p-s)}{s(p-q)}, & \text{if } 0 < s < q < r \leq p, \\ \dfrac{p(q-s)}{s(p-q)}, & \text{if } 0 < s < r \leq q. \end{cases}$$

This result improves the result of R. Fukuda [1]. This inequality is very useful in many problems of probability theory.

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Interactive Mathematics Based on Technology "Mathematicos"

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The report describes the possibilities of using technology "Mathematicos" to create interactive lessons with built-in online teacher, accompanying the student's actions in solving problems, performing detailed, step-by-step assistance. We propose technics for learning lessons, solving problems and exercises through mediation and under control of the virtual teacher. We show that incorporation of such machine teacher as a fully-fledged technological participant in the process of everyday learning provides a significant improvement in the individual indicators of students.

e-Infrastructure for Research and Education in Georgia

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The status of network and computing infrastructure and available services for research and education community of Georgia are presented. The network services provided by

GRENA to universities and research institutions are: Internet and GEANT connectivity, eduroam – educational roaming, eduGAIN – authentication and authorization, CERT Computer Emergency Response Team, Intrusion Detection/Prevention systems IDS/IPS, etc.

GRENA computational resources consists of Grid (distributed computing) GE-01-GRENA site included in European Grid infrastructure and Virtualization facility. Computing resources used by the research teams working in weather research and forecasting, climate modelling, life sciences and computational chemistry.

Currently development *e*-Infrastructure and services is performed in the framework of European Commission projects: EaPConnect, GN4-2 Research and Education Networking – GÉEANT and VRE for regional Interdisciplinary communities in Southeast Europe and the Eastern Mediterranean. Results of these research and developments are briefly presented.

On the Exactness of the Unknown Density Approximation by a Nonparametric Estimate Constructed by Conditionally Independent Observations

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We consider a two-component stationary (in a narrow sense) sequence $\{\xi_i X_i\}_{i\geq 1}$, $\{X_i\}_{i\geq 1}$ is a conditionally independent sequence, while $\{\xi_i\}_{i\geq 1}$ is a sequence of equally distributed independent random variables. $\xi_i = \sum\limits_{k=1}^{m} \alpha_k I_{(\xi_i=\alpha_k)}$, $P(\xi_i = \alpha_k) = P_k$, $k = \overline{1,m}$. The distributions $P_{(\xi_i=\alpha_k)=\alpha_k}$ have unknown densities $f_k(x)$, $k = \overline{1,m}$. The exactness of the approximation of the unknown density by the estimate $\widehat{f}(x) = \sum\limits_{k=1}^{m} P_k f_k(x)$ obtained by conditionally independent observations $\widehat{f}_n(x)$ is established $x_1, x_2, \ldots, x_n$.

On Consistent Estimator of a Useful Signal in Ornstein–Uhlenbeck Model in $C[-l,l[$

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We consider a transmission process of a useful signal in Ornstein–Uhlenbeck stochastic model in $C[-l,l[$ defined by the stochastic differential equation

$$d\Psi(t,x,\omega)=\sum_{n=0}^{2m}A_n\,\frac{\partial^n}{\partial x^n}\,\Psi(t,x,\omega)\,dt+\sigma\,dW(t,\omega)$$

with initial condition:

$$\Psi(0,x,\omega)=\Psi_0(x)\in FD^{(0)}[-l,l[\,,$$

where $m\geq 1$, $(A_n)_{0\leq n\leq 2m}\in\mathbb{R}^+\times\mathbb{R}^{2m-1}$, $((t,x,\omega)\in[0,+\infty[\,\times[-l,l[\,\times\Omega)$, $\sigma\in\mathbb{R}^+$, $C[-l,l[$ is Banach space of all real-valued bounded continuous functions on $[-l,l[$, $FD^{(0)}[-l,l[\subset C[-l,l[$ denotes the class of all real-valued bounded continuous functions on $[-l,l[$ whose Fourier series converges to himself everywhere on $[-l,l[$, $(W(t,\omega))_{t\geq 0}$ is a Wiener process and $\Psi_0(x)$ is an useful signal.

By use a sequence of transformed signals $(Z_k)_{k\in\mathbb{N}}=(\Psi(t_0,x,\omega_k))_{k\in\mathbb{N}}$ at moment $t_0>0$, consistent and infinite-sample consistent estimates of the useful signal Ψ_0 is constructed under assumption that parameters $(A_n)_{0\leq n\leq 2m}$ and σ are known.

Pareto Efficiency and Achievable Distribution

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It is interesting to answer the following question, how successfully achieves concurrent market the Pareto efficiency. The distribution method is a Pareto efficient if there is alternative distribution where no one will suffer, but even certain number of customers condition will be improved. Pareto-efficiency can be described as the distribution, where 1. there is no ability to improve all's condition; or 2. there are no ways to improve condition of any individual without worsening other individuals condition. Distribution is achievable, if the total amount of consumptions of each product is equal to its total number [1]: Here is some equation

$$\begin{aligned} x_A^1 + x_B^1 &= \omega_A^1 + \omega_b^1, \\ x_A^2 + x_B^2 &= \omega_A^2 + \omega_B^2. \end{aligned}$$

If we use differential calculus for description of Pareto efficiency distribution, the formula of maximization will get the following form:

$$\max_{x_A^1, x_A^2, x_B^1, x_B^2} U_A(x_A^1, x_A^2),$$

when

$$u_B(x_B^1, x_B^2) = \bar{u}, \;\; x_A^1 + x = \omega^1; \;\; x_A^2 + x_B^2 = \omega^2.$$

we can solve this problem by composing Lagrangian, which will have such form:

$$L = u_A(x_A^1, x_B^2) - \lambda\big(u_B(x_B^1, x_B^2) - \overline{u}\big) - \mu(x_A^1 + x_B^1 - \omega^1) - \mu_2(x_A^2 + x_B^2 - \omega^2).$$

If we differentiate for each commodity and make some kind of transformations we will get:

$$\begin{aligned} MRS_A &= \frac{\partial u_A/\partial x_A^1}{\partial u_A/\partial x_A^2} = \frac{\mu_1}{\mu_2}\,, \\ MRS_A &= \frac{\partial u_B/\partial x_B^1}{\partial u_B/\partial x_B^2} = \frac{\mu_1}{\mu_2}\,. \end{aligned}$$

Keywords: Pareto efficiency, distribution method, substitution marginal rate, Lagrangian.

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On the Norm Estimates of Fourier Integrals Summability Means in Weighted Grand Lebesgue Space on the Axis

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Let $1 < p < \infty$, $\theta > 0$. Weighted grand Lebesgue space on $(-\infty,\infty)$ is defined as

$$\mathcal{L}_W^{p),\theta}(R^1) = \Bigg\{ f:\ \|f\|_{\mathcal{L}_W^{p),\theta}} = \sup_{0<\varepsilon<p-1} \bigg(\varepsilon^{\theta} \int\limits_{-\infty}^{+\infty} |f(x)|^{p-\varepsilon} w(x) \big(\sqrt{1+|x^2|}\big)^{-\alpha\varepsilon}\, dx \bigg)^{\frac{1}{p-\varepsilon}} < +\infty \Bigg\},$$

where $\alpha > p$. For definition see [1, p. 800].

We assume that the weight w belongs to the well-known Cesàro Muckenhoupt class.

The goal of our talk is to present the uniform estimates in $\mathcal{L}_W^{p),\theta}(-\infty,\infty)$ of the norms Fourier integral's Cesàro and Abel–Poisson summability means. The norm convergence problems will be treated in some subspaces of aforementioned spaces. For the analogous results in weighted Lebesgue spaces with variable exponent, we refer the readers to [2].

MSC 2000: 42B20, 47B38.

Keywords: Fourier integrals, weighted grand Lebesgue space.

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On Stochastic Differential Equation Driven by the Cylindrical Wiener Process

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We consider the problem of existence and uniqueness of the solution of the stochastic differential equation driven by the cylindrical Wiener process. According to our approach, We construct the corresponding stochastic differential equation in the space of the generalized random elements, find the generalized solution as a generalized random process and the question of existence of the solution of the main equation in a Banach space we reduce to the well known problem of decomposability of the generalized random elements (see [1]).

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Liouville Type Theorems for First Order Singular Systems

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The present work deals with the interpretation of uniqueness theorem of the theory of analytic functions, its generalization and some applications.

Some Existence and Uniqueness Results for Fractional Differential Equations with Caputo–Fabrizio Derivative

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Compared with integer order, a significant feature of a fractional order differential operator is its nonlocal behavior. In other words, the future state of a process described by fractional derivative depends on its current state as well as its past states. Therefore, by making use of differential equations of arbitrary order, one can describe the memory and hereditary properties of various important materials and systems. So, in recent years fractional differential equations have been paid of great interest and there appeared new areas for applications of initial and boundary value problems of such equations. There are different definitions for fractional derivative and recently some new definitions for derivative of arbitrary order have been updated.

The new Caputo–Fabrizio fractional derivative, with no singular kernel, was defined as the following

Definition 1. For a function $f\in H^1(a,b)$ the Caputo–Fabrizio derivative of fractional order $\alpha\in[0,1]$ is defined as

$$D_t^\alpha(f(t))=\Big(\frac{M(\alpha)}{1-\alpha}\Big)\int\limits_a^t f'(x)\exp\Big[-\alpha\,\frac{t-x}{1-\alpha}\Big]\,dx,$$

such that $M(\alpha)$ is a normalization function under the conditions $M(0)=M(1)=1$.

Definition 2. The fractional arbitrary order integral of a function $f\in H^1(a,b)$ is defined as follows

$$I_\alpha^t(f(t))=\frac{2(1-\alpha)}{(2-\alpha)\,M(\alpha)}\,f(t)+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int\limits_0^t f(s)\,ds,\quad t\geq 0.$$

Caputo–Fabrizio fractional derivative has many significant properties such as its ability in describing matter heterogeneities and configurations with different scales. The existence of solutions for nonlinear differential equations has been studied using the techniques of nonlinear analysis such as fixed point results [1, 2] and stability.

In this paper, we consider the fractional differential equations via the new fractional derivative of Caputo–Fabrizio and study some existence and uniqueness results for such equations.

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Teaching a Concept

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We use concepts for teaching any subject, often without even realizing if student adequately perceives them or not. Usually concepts are the foundation on which further knowledge is built, so the success of the teaching process is largely dependent on the whether the perception of concept is correct or not. The author discusses different methods of teaching (specificity, generalization) and common mistakes that are made during the teaching process, such as development of inappropriate perception in the mind of student caused by limited number of samples; Analysis of sample schemes of hierarchical characteristic features of concepts (cause and effect), specifics of using these schemes and based on them the possible problems related to making alternative definitions; A sample of Concept Map isexamined, such as concepts' visible succession scheme and didactic peculiarities of its usage.

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Simulation of Air Pollutant Distribution Over the Caucasus on the Bases of WRF-Chem Model

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This study evaluates the spatiotemporal distribution of dust over Caucasus region using WRF-Chem model. The model runs in operational mode on GRENA's High Performance Computing (HPC) center. It makes 3-days forecast of mineral dust distribution and creates maps for the distribution of the particles with 2.4um in diameter over the Caucasus domain for the 4 different values of height with 1 hour pitch. For the initial and boundary conditions the model uses Global Forecast System (GFS) model data.

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Software of Distributed Computing Network Monitoring and Analysis

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The brief description of programming for controlling and analyzing information about functioning distributed computing network is presented. Operating in the menu mode an administrator and engineer are consider a network temporary state, tracing map of token and statistic of commands, used by users of the network. Using screen menu mode of operation, the user can copy, rename, restore, find and change phrases, to print files. The program could be easily transferred to any computer of CP/M operation system. Operating in the menu mode, the user can specify a node architecture, the number of ports to be serviced, and give parameters for the initial adjusting of the ports for specific equipment. The user can set up a connection with the nodes hooked up to the computer, load the programs prepared for the further debugging, look through, copy and modify the files with the net software.

A Characterization of Bigroups of Operations

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By analogy of bilattices [1]–[3] we consider the concepts of a bisemigroup, a bimonoid, a De Morgan bisemigroup and a bigroup.

A bimonoid $Q(\cdot, \circ)$ with identity elements 0 (for operation $\cdot$) and 1 (for operation $\circ$) is called a bigroup, if for every $x \in Q$:

$$0 \circ x = 0, \quad 1 \cdot x = 1,$$

and the following conditions are valid:

(a) $Q \setminus \{1\}$ is a group with an identity element 0 under the multiplication $\cdot$;

(b) $Q \setminus \{0\}$ is a group with an identity element 1 under the multiplication $\circ$;

A bigroup of order > 3 is called non-trivial.

The set $O_p^{(2)}Q$ of all binary operations on the set Q is a bimonoid under the following operations:

$$f \cdot g(x, y) = f(x, g(x, y)), \tag{1}$$
$$f \circ g(x, y) = f(g(x, y), y), \tag{2}$$

in which the identity elements are the identical operations δ_2^2 and δ_2^1,where $\delta_2^1(x, y) = x$, and $\delta_2^2(x, y) = y$ for all $x, y \in Q$. Any subset $S \subseteq O_p^{(2)}Q$ which is closed under these two operations is called a bisemigroup of operations (on the set Q). The bisemigroup of operations (on the set Q) is called a bimonoid of operations (on the set Q) if it contains the identical operations δ_2^1 and δ_2^2.

The bimonoid S of operations (on the set Q) is a bigroup, if both of the following conditions are valid:

(c) $S \setminus \{\delta_2^1\}$ is a group with an identity element δ_2^2 under the multiplication (1);

(d) $S \setminus \{\delta_2^2\}$ is a group with an identity element δ_2^1 under the multiplication (2);

Such bigroup is called a bigroup of operations (on the set Q).

We give a local characterization of bigroups of operations.

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A Characterization of the Belousov Variety

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The concept of Belousov quasigroup and Belousov variety are introduced and it is proved that the nontrivial Belousov quasigroup has at least five elements. The variety of Belousov quasigroups is a subvariety of the Mikado variety. The Belousov variety has a solvable word problem and is congruence-permutable. Every Belousov quasigroup of prime order is a simple algebra. However the solution of the following problem is open: to which loops are Belousov quasigroups isotopic?

Numerical Solution of the Non-Linear Biharmonic Equation for Different Boundary Conditions

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Biharmonic equation plays an important role in different scientific disciplines, but it is difficult to solve due to the existing fourth order derivatives. They arise in several areas of mechanics such as two dimensional theory of elasticity and the deformation of elastic and elasto-plastic plates. The mathematical model related to elasto-plastic bending plate can be written by the non-linear biharmonic equation.

Various approaches for the numerical solution of biharmonic equation have been considered in the literature over several decades. In this study, bending problem of the plate which is an elasto-plastic and homogeneously isotropic incompressible features is studied. The main purpose of this work is obtain numerical solution of the nonlinear biharmonic equation corresponding to the bending problem of the elasto-plastic plate with different boundary conditions by the finite difference method.

The study is contained that mathematical model of bending problem of non-linear plate, its finite difference approach and numerical solution. To check the correctness of the numerical solutions, a test function which is proved conditions of the problem is used.

The numerical results obtained by the computational experiments have interesting aspects of both mathematical and engineering points of view. The presented numerical examples show the effectiveness of the given approach.

Study on Meromorphic Functions Based on Subordination

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This paper investigates sharp coefficient bounds, integral representation, extreme point, and operator properties of a certain class associated with functions which are meromorphic in the punctured unit disk.

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Amply Weak e-Supplemented Modules

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In this work, amply weak e-supplemented modules are defined and some properties of these modules are investigated. It is proved that every factor module and every homomorphic image of a amply weak e-supplemented module are amply weak e-supplemented.

Definition. Let M be an R-module and $U \leq M$. If for every $V \leq M$ such that $M = U + V$, U has a weak supplement V' with $V' \leq V$, we say U has ample weak supplements in M. If every essential submodule of M has ample weak supplements in M, then M is called an amply weak e-supplemented module.

Lemma 1. *Let M be an R-module. If every submodule of M is weakly e-supplemented, then M is amply weak e-supplemented.*

Lemma 2. *Let R be any ring. Then every R-module is weakly e-supplemented if and only if every R-module is amply weak e-supplemented.*

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Key words and phrases. Essential submodules, small submodules, radical, supplemented modules..

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ET-Small Submodules

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Let M be an R-module and $T \leq M$. A submodule K of M is called ET-small in M, denoted by $K \ll_{ET} M$, in case for any essential submodule X of M, $T \leq X + K$ implies that $T \leq X$. In this work, some properties of these submodules are investigated.

Proposition 1. *Let M be an R-module and $K \leq M$. Then $K \ll_{EM} M$ if and only if $K \ll_e M$.*

Proposition 2. *Let M be an R-module and $K \leq T \leq M$. Then $K \ll_{ET} M$ if and only if $K \ll_e T$.*

Proposition 3. *Let M be an R-module and $T, K_1, K_2 \leq M$. If $K_1 \ll_{ET} M$ and $K_2 \ll_{ET} M$, then $K_1 + K_2 \ll_{ET} M$.*

2010 Mathematics Subject Classification. 16D10, 16D80.

Key words and phrases. Essential sumbodules, small submodules, T-small submodules, e-small submodules.

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A Generalization of a Relation Between Means

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Let $n > 1$, $x_1, \dots, x_n$ be natural numbers, $x_1 + \cdots + x_n = L$, let also $L = nq + r$, where q is a natural number and $0 \le r < n$ is an integer. Then we have

$$x_1 \cdots x_n \le (1+q)^r q^{n-r}$$

and the equality takes place if and only if $|x_i - x_j| \in \{0, 1\}$ for $i, j = 1, \dots, n$.

In case when L is not divisible on n (i.e. $r \neq 0$) this inequality is a refinement of classic mean-arithmetic–mean-geometric inequality. A verification of this, even for cases $n = 2$ or $n = 3$, would be a good exercise for the pupils interested in mathematics.

The talk is based on [1].

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About the Mathematical Model of Progression and Treatment of Autoimmune Diseases

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The model-based investigation of diseases is complex and evolving field. We herein report about our attempt to establish the mathematical model and its computational implementation for autoimmune diseases. Mathematical models of immune mediated disorders provide an analytic framework in which we can address specific questions concerning disease immune dynamics and the choice of treatment. Such models are actively reported in the field of tumor immunology and immunotherapy by de *Phillis et al* [1], [2]. At the last conference we presented mathematical model of rheumatoid arthritis. We generalized the mode for all autoimmune diseases and consider rheumatoid arthritis as a particular case

$$\begin{cases} \dfrac{dJ(t)}{dt} = r_0 J(t)(1 - b_0 J(t)) - a_0 J(t)(B(t) - B^{norm}), \\ \dfrac{dB(t)}{dt} = r_1 B(t)(1 - b_1 B(t)) + c_1 B(t) T_h(t) - d_1 B(t) T_{reg}(t), \\ \dfrac{dT_h(t)}{dt} = r_2 T_h(t)(1 - b_2 T_h(t)), \\ \dfrac{dT_{reg}(t)}{dt} = s_2 - d_2 T_{reg}(t). \end{cases}$$

Now we present generalized and improved model. Improvement includes estimation of model coefficients and the software that solves the Cauchy problem for this system and visualizes the obtained solution is developed. We also add drug component to the model. Solutions can be obtained for the case of each individual patient to decipher the disease progress and the absence of disease.

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Numerical Computation of the Kirchhoff type Nonlinear Static Beam Equation by Iterative Method

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Let us consider the nonlinear beam equation

$$u''''(x) - m\Big(\int_0^l {u'}^2(x)\,dx\Big)u''(x) = f(x,u(x),u'(x)), \quad 0 < x < l, \tag{1}$$

with the conditions

$$u(0) = u(l) = 0, \quad u''(0) = u''(l) = 0. \tag{2}$$

Here $u = u(x)$ is the displacement function of length l of the beam subjected to the action of a force given by the function $f(x,u(x),u'(x))$, the function $m(z)$,

$$m(z) \ge \alpha > 0, \quad 0 \le z < \infty, \tag{3}$$

describes the type of a relation between stress and strain.

The questions of the solvability of problem $(1),(2)$ is studied in [1], while the problem construction of numerical algorithms and estimation of their accuracy is investigated in [2], [3]. In the present paper, in order to obtain an approximate solution of the problem $(1),(2)$ an approach is used, which differs from those applied in the above-mentioned references. It consists in reducing the problem $(1),(2)$ by means of Green's function to a nonlinear integral equation, to solve with we use the iterative process. The condition for the convergence of the method is established and numerical realization is obtained. The algorithm has been approved tests and the results of recounts are represented in graphics.

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Numerical Calculations of the Timoshenko Type Dynamic Beam Nonlinear Integro-Differential Equation

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Let us consider the nonlinear equation

$$u_{tt}(x,t)+\delta u_t(x,t)+\gamma u_{xxxxt}(x,t)+\alpha u_{xxxx}(x,t)-\bigg(\beta+\rho\int\limits_0^L u_x^2(x,t)\,dx\bigg)u_{xx}(x,t)$$
$$-\sigma\bigg(\int\limits_0^L u_x(x,t)u_{xt}(x,t)\,dx\bigg)u_{xx}(x,t)=0,\ \ 0<x<L,\ \ 0<t\le T, \quad (1)$$

with the initial boundary conditions

$$\begin{gathered} u(x,0)=u^0(x), \quad u_t(x,0)=u^1(x), \\ u(0,t)=u(L,t)=0, \quad u_{xx}(0,t)=u_{xx}(L,t)=0. \end{gathered} \quad (2)$$

Here α, γ, ρ, σ, β and δ are given constants, among which the first four are positive numbers, while $u^0(x)\in W_2^2(0,L)$ and $u^1(x)\in L_2(0,L)$ are given functions such that $u^0(0) = u^1(0) = u^0(L) = u^1(L) = 0$. We suppose that there exits a solution $u(x,t) \in W_2^2((0,L) \times (0,T))$ of problem (1), (2). The equation (1) obtained by J. Ball [1] using the Timoshenko theory describes the vibration of a beam. Solution of problem (1), (2) is founded by means of an algorithm, the constituent parts of which are the Galerkin method, an implicit symmetric difference scheme and Jacobi iterative method (see [2]). For approximate solving boundary value problem (1), (2) the some of programs in algorithm language Maple is composed and many numerical experiments are carried out.

The author express hearing thanks to Prof. J. Peradze for his active help in problem statement and solving.

References

[1] J. M. Ball, Stability theory for an extensible beam. *J. Differential Equations* **14** (1973), 399–418.

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A Difference Scheme for a Nonlinear Integro-Differential Wave Equation

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We consider the initial boundary value problem

$$u_{tt}(x,t) + \delta u_t(x,t) + \gamma u_{xxxxt}(x,t) + \alpha u_{xxxx}(x,t) - \left(\beta + \rho \int_0^L u_x^2(x,t)\,dx\right) u_{xx}(x,t)$$

$$-\sigma\left(\int_0^L u_x(x,t) u_{xt}(x,t)\,dx\right) u_{xx}(x,t) = 0, \quad 0 < x < L, \ \ 0 < t \le T, \tag{1}$$

$$\begin{gathered} u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x), \\ u(0,t) = u(L,t) = 0, \quad u_{xx}(0,t) = u_{xx}(L,t) = 0, \end{gathered} \tag{2}$$

where α, γ, ρ, σ, β and δ are the given constants, among which the first four are positive numbers, while $u^0(x)$ and $u^1(x)$ are given sufficiently smooth functions.

Equation (1) obtained by J. M. Ball [1] using the Timoshenko theory describes the vibration of the beam. To approximate the solution of problem (1), (2) with respect to a time variable a difference scheme is used and its error is estimated.

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Dirichlet Problem in the Weighted Spaces $L^1(\rho)$

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Let T be a unit circle in the complex plane z, and let D^+ and D^- be the interior and exterior domains, respectively bounded by the curve T. Also, $\rho(t) = \prod\limits_{k=1}^{m} |t - t_k|^{\alpha_k}$, where $t_k \in T$ and α_k, $k = 1, \dots, m$, are arbitrary real numbers.

The Dirichlet boundary value problem is investigated in the following setting:

Problem D. *Let f be a real-valued, measurable on T function from the class $L^1(\rho)$. Determine an analytic in D^+ function $\Phi(z)$ to satisfy the boundary condition:*

$$\lim_{r \to 1-0} \big\| \operatorname{Re} \Phi(rt) - f(t) \big\|_{L^1(\rho_r)} = 0.$$

With the famous transformation this problem is reduced to Riemann boundary value problem which is investigated in the work [1].

1. On the general solution of the homogeneous problem

(a) *If $N > -1$, then the general solution of the homogeneous Problem D can be represented in the form:*

$$\Phi_0(z) = \frac{1}{\Pi(z)} (c_0 z^N + c_1 z^{N-1} + \dots + c_N),$$

where numbers $\{c_l\}_{l=0}^{N}$ satisfy the following condition:

$$(-1)^{N+1}\overline{c_l}\prod_{k=1}^{m} t_k^{n_k} = c_{N-l}, \quad l = 0, 1, \ldots, N.$$

(b) *If $N \le -1$, then the homogeneous problem has only trivial solution.*

2. On the general solution of Dirichlet problem

The general solution is given in the form $\Phi(z) = \Omega(f, z) + \Phi_0(z)$, where

$$\Omega(f, z) = \frac{1}{\pi i \Pi(z)}\bigg(\int_T \frac{f(t)(t^N + z^N)\Pi(t)}{t^N(t-z)}\,dt - \int_T \frac{f(t)\Pi(t)}{t^{N+1}}\,dt\bigg).$$

Besides, in the case $N \le -1$ there are given necessary and sufficient conditions for solvability of the problem.

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ენობრივი რესურსებითა და ტექნოლოგიებით ქართული ენის სრული უზრუნველყოფის ერთიანი გეგმის პირველი კონცეპტუალური მონახაზი

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია, შალვა მალიძე

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [1] მიზანია ქართული მოაზროვნე, მოსაუბრე და მთარგმნელი

სისტემის აგება, რაც აუცილებლობით ითხოვს და, აგრეთვე, თითქმის უტოლდება ქართული უნივერსალური ჭკვიანი კორპუსის სრული ვერსიის აგებას.

ამასთან, ზემოაღნიშნული ენობრივი რესურსებითა და ტექნოლოგიებით ქართული ენის სრულ უზრუნველყოფას ნიშნავს. ამგვარად, იმ გამოცდილების გათვალისწინებით, რაც 2012–2017 წლებში პროექტის „ქართული ენის ტექნოლოგიური ანბანი" ფარგლებში წარმოებული კვლევებითა და ამ კვლევების არაუმნიშვნელო შედეგებით დაგროვდა [2], [3], მოხსენებისას წარმოვადგენთ ენობრივი რესურსებითა და ტექნოლოგიებით ქართული ენის სრული უზრუნველყოფის ერთიანი გეგმის პირველ კონცეპტუალურ მონახაზს. აქვე შევნიშნავთ, რომ ეს კონცეპტუალური მონახაზი სრულ თანმთხვევაშია [2]-[5]:

1. სახელმწიფო ენის შესახებ საქართველოს კანონის 37-ე მუხლის მე-3 პუნქტთან, რომლის თანახმადაც **„სახელმწიფო ენის ერთიანი პროგრამა"** უნდა ითვალისწინებდეს **„ქართული ენის სრულ ტექნოლოგიურ უზრუნველყოფას"**;
2. საქართველოს მეცნიერებათა ეროვნული აკადემიის აკადემიური საბჭოს 2015 წლის 29 მაისის № 28 დადგენილებით შექმნილ **„ქართული ტექსტური ბაზების ტექნოლოგიური უზრუნველყოფის ძირითადი პრინციპების შემმუშავებელი საკონსულტაციო საბჭოს"** მიზნებთან.

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[3] კ. ფხაკაძე, მ. ჩიქვინიძე, გ. ჩიჩუა, დ. კურცხალია, ა. მასხარაშვილი, ქართული ენის ლოგიკური გრამატიკა: საფუძვლები და გამოყენებები - I ნაწილი, ჟურნალი „ქართული ენა და ლოგიკა", № 9, 2015-2016, 4-128.

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ქართული ჭკვიანი კორპუსი - უკვე მიღწეული შედეგებისა და მისაღწევი მიზნების ზოგადი მიმოხილვა

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია, ინეზა ბერიაშვილი, შალვა მალიძე

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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სტუ ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [1] ორწლიანი ქვეპროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ ფარგლებში შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის №AR/122/4-105/14 საგრანტო მხარდაჭერით მიმდინარე კვლევებით შემუშავდა ქართული ინტელექტუალური ვებ-კორპუსი, სხვა სიტყვებით, ქართული ჭკვიანი კორპუსი [2], რომელიც, მიუხედავად იმისა, რომ ფასდება ქართული (უნივერსალური) ჭკვიანი კორპუსის აგების მიმართულებით გადადგმულ ჯერ მხოლოდ პირველ ნაბიჯად უკვე არის:

1. ქართული ენის ყველაზე მოცულობითი კორპუსი. ეს ყოველწუთიერ ავტომატურად განვითარებადი კოპრუსი ჯამურად უკვე შეიცავს 258 მილიონ სიტყვა-ტოკენზე მეტს, რომელთანგან 4 მილიონზე მეტი განსხვავებულია;
2. ტექნოლოგიურად ყველაზე უფრო მეტად მხარდაჭერილი ქართული კორპუსი. გარდა კორპუსის ავტომატურად განმავითარებელი ინტელექტუალური ინსტრუმენტებისა, მასში ჩადგმული სახით უკვე ფუნქციონირებს მთელი რიგი ქართული უნიკალური სისტემები ენობრივი ტექნოლოგიების ისეთი ძირითადი მიმართულებებით, როგორიცაა ტექსტის ანალიზი, მეტყველების დამუშავება, ავტომატური თარგმანი, დიალოგური ურთიერთობები, ერთენოვანი/მრავალენოვანი კომუნიკაცია და სწავლა-სწავლება.

ამგვარად, მოხსენებისას ზოგადად მიმოვიხილავთ ქართულ ჭკვიან კოპრუსსა და მასში ჩადგმულ ტექნოლოგიურ სისტემებს. ასევე, მოკლედ წარმოვადგენთ ქართული ჭკვიანი კორპუსის თეორიულ საფუძვლებსა და მისი შემდგომი განვითარების მიზნებსა და მიმართულებებს [2], [3].

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პროექტი „ქართული ენის ტექნოლოგიური ანბანი" ანუ ქართული ჭკვიანი კორპუსის საცდელი ვერსიიდან ქართული (უნივერსალური) ჭკვიანი კოპრუსის სრულ ვერსიამდე

კონსტანტინე ფხაკაძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია, შალვა მალიძე

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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სტუ ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი" [1] ქვეპროექტით „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ" [2] შემუშავდა ქართული ინტელექტუალური ვებ-კორპუსის, სხვა სიტყვებით, ქართული ჭკვიანი კორპუსის პირველი საცდელი ვერსია. ამავე გრძელვადიანი პროექტის კიდევ ქვეპროექტი „ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება" [3] მიზნად ისახავს უკვე არსებული ქართული ჭკვიანი კორპუსის გაფართოებისა და გაძლიერების გზით ქართული (უნივერსალური) ჭკვიანი კორპუსის საცდელი ვერსიის აგებას. ამასთან, პროექტით „ქართული ენის ტექნოლოგიური ანბანი" დასახული მიზნის მიღწევა ანუ ქართული მოაზროვნე, მოსაუბრე და მთარგმნელი სისტემის აგება აუცილებლობით ითხოვს და, აგრეთვე, თითქმის უტოლდება ქართული უნივერსალური ჭკვიანი კორპუსის სრული ვერსიის აგებას.

ამგვარად, მოხსენებისას მიმოვიხილავთ ქართული ენის ლოგიკური გრამატიკის ფარგლებში შემუშავებულ იმ გზებსა და მეთოდებს, რაც ჩვენ მიგვიყვანს ქართული ჭკვიანი

კორპუსის უკვე არსებული საცდელი ვერსიიდან ამავე კორპუსის სრულ ვერსიამდე, რაც, მთლიანობაში, თითქმის უზრუნველყოფს როგორც გრძელვადიანი პროექტით „ქართული ენის ტექნოლოგიური ანბანი“ დასახული მიზნის მიღწევას, ისე ქართული ენის დაცვას ციფრული კვდომის საფრთხისგან.

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ქართული ჭკვიანი კორპუსი - თითქმის სრულად ადაპტირებული ქართული საინტერნეტო სივრცის ფორმირებისკენ გადადგმული მნიშვნელოვანი ნაბიჯი

კონსტანტინე ფხაკაძე, დავით კურცხალია, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, შალვა მალიძე

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ინტელექტუალური კორპუსისაკენ“ [1] ფარგლებში შემუშავდა ქართული ინტელექტუალური ვებ-კორპუსი, სხვა სიტყვებით, ქართული ჭკვიანი კორპუსი, რომელიც თავის თავში აერთიანებს ქართული საიტერნეტო პრესის მეტ წილს.

ამასთან, იმის ხარჯზე კორპუსი ნაწილობრივ უკვე აღჭურვილია ჩვენს მიერვე შემუშავებული საცდელი სახით მომუშავე ქართული კომპიუტერული ტვინით (ანუ ქართული ტექსტების მაანალიზებელი, მაგენერირებელი და მართლწერი სისტემებით), ყელით (ანუ ქართული ელექტრონული ტექსტების გამახმოვანებელი ანუ მკითხველი სისტემებით), ყურით (ანუ ქართული მეტყველების ამომცნობი სისტემითა და ქართული ხმოვანი მართვის მოდულით), მთარგნელით (ანუ ქართულიდან და ქართულზე ხმიდან ხმაზე და ტექსტიდან ტექსტზე მთარგნელი სისტემებით) მომხმარებელს საშუალებას აძლევს: 1. ხმით ანუ მოსმენით მიიღოს ინფრომაცია კორპუსის სტატიებიდან (მათი რაოდენობა 2 მილიონს აჭარბებს) მათზე მიდგმული ჩვენს მიერვე შემუშავებული ხმოვანმართვიანი მკითხველი სისტემით; 2. ამ სტატიებში აწარმოოს ჩვენს მიერ ეგრეთ წოდებული ხმოვანი ძებნა და დაძებნილი ფრაზით ან ამ ფრაზის უცხოენოვანი თარგმნით აწარმოოს ხმოვანი ბრძანებითვე წამოწყებული ძებნა როგორც ქართულ ისე შესაბამის უცხოენოვან გუგლსა და ვიკიპედიაში.

ეს ყველაფერი გასაგებს ხდის, რომ ქართული ჭკვიანი კორპუსი ყველაფერ დანარჩენთან ერთად, ასევე უნდა შეფასდეს როგორც პირველი საცდელი ნაბიჯის [2] შემდეგ გადადგმული მნიშვნელოვანი ნაბიჯი თითქმის სრულად ადაპტირებული ქართული საინტერნეტო სივრცის ფორმირებისკენ, რაც, თავის მხრივ, კორპუსს მნიშვნელოვან პრაქტიკულ ღირებულებებს სძენს ქართველი უსინათლოების ქართულ საინტერნეტო სივრცეში უფრო მეტი მოცულობით ინტეგრირების თვალსაზრისით.

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საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების ცენტრის პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ ორწლიანი ქვეპროექტის „კიდევ ერთი ნაბიჯი მოსაუბრე ქართული თვითგანვითარებადი ინტელექტუალური კორპუსისაკენ“ [1] ფარგლებში შემუშავდა ქართულ ჭკვიან კორპუსში ჩადგმული სახით ფუნქციონირებადი ქართული (ჭკვიანი) ფურცელი, რომელზეც დაყრდნობითაც აიგო მასში ასევე ჩადგმული სახით ფუნქციონირებადი ქართული ერთენოვანი და მრავალენოვანი საინტერნეტო და სამობილურო სისტემები. ამასთან, ეს სისტემები მომხმარებლებს საშუალებას აძლევს თავისი ქართული ჭკვიანი ფურცლიდან სხვა ქართულ ჭკვიან ფურცელს გადაუგზავნოს:

1. ქართული ტექსტუალური შეტყობინება ან/და ქართული ტექსტუალური შეტყობინების ქართულადვე სინთეზირებული ანუ გახმოვანებული ვერსია ან/და ქართული ტექსტობრივი შეტყობინების უცხოენოვანი თარგმანი. - უახლოეს მომავალში იგეგმება ამ ფუქნციონალის გაფართოება ქართული ტექსტუალური შეტყობინების უცხოენავანი თარგმანის გახმოვანებული ვერსიების გადაგზავნის შესაძლებლობებით.
2. ქართული ან/და უცხოენოვანი ხმოვანი შეტყობინება. - სამომავლოდ ვგეგმავთ ამ ფუნქციონალის გაფართოებას როგორც ქართული, ისე უცხოენოვანი ხმოვანი შეტობინებების მინიშნებულ ენაში თარგმნილი ვერსიის გადაგზავნის შესაძლებლობებით.

ამგვარად, გასაგები ხდება, რომ ქართული ჭკვიანი კორპუსი, ყველაფერ დანარჩენთან ერთად, ასევე უნდა შეფასდეს როგორც ქართული ერთენოვანი და მრავალენოვანი საინტერნეტო და სამობილურო საკომუნიკაციო სისტემების აგების მიმართულებით გადადგმულ მნიშვნელოვან ნაბიჯად. ამგვარად, ცხადია, როგორც ის, რომ რომ ეს ყველაფერი ქართულ ჭკვიან კორპუსს მნიშვნელოვან პრაქტიკულ ღირებულებებს სძენს, ასევე ისიც, რომ ამით ქართულ ენა ციფრულ სამყაროში უფრო ნაყოფიერად გამოყენებადი ანუ უფრო „კომფორტული“ ხდება, ვიდრე ის მანამდე იყო.

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გამოიცა საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრის სამეცნიერო-საგანმანათლებლო ჟურნალის „ქართული ენა და ლოგიკა" დამატების სახით, (2017), 4-320.

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აფხაზური ენის კორპუსი - პროექტის „ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება" ერთ-ერთი მნიშვნელოვანი მიზანი

კონსტანტინე ფხაკაძე, შალვა მალიძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია

საქართველოს ტექნიკური უნივერსიტეტი, ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრი, თბილისი, საქართველო

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საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების ცენტრის გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი" [1] ორწლიანი ქვე-პროექტის „ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება" [2] ერთ-ერთი მეტად მნიშვნელოვანი მიზანია აფხაზური ენის თვითგანვითარებადი ანუ ავტომატურად განვითარებადი ვებ-კორპუსის შემუშავება, რაც, ცხადია, სხვაგვარად გარდა თუ არა აფხაზური ენის ციფრული კვდომის საფთხისგან დაცვისთვის აუცილებელი მოცემულობა, ვერ შეფასდება. ამასთან, სადოქტორო პროექტის მიზანია ქართული ჭკვიანი კორპუსისა და მასში ჩადგმული ტექნოლოგიური სისტემების გაფართოებისა და გაძლიერების გზით ქართული უნივერსალური ჭკვიანი კორპუსის საცდელი ვერსიის აგება, რაც, ცხადია, კიდევ უფრო გაზრდის ქართული ენის ციფრული კვდომის საფრთხისგან დაცულობის ხარისხს [3]. ეს ყველაფერი, მთლიანობაში, გასაგებს ხდის სადოქტორო პროექტის მეტად მაღალ მნიშვნელობას საქართველოს სახელმწიფო

ენების - ქართულისა და აფხაზურის ციფრული კვდომის საფრთხისგან დაცვის ერთიანი ქართული სახელმწიფოებრივი მიზნებისა და პასუხისმგებლობების ჭრილში.

ამგვარად, მოხსენებისას განსაკუთრებულ ყურადღებას გავამახვილებთ პროექტით ასაგებად დაგეგმილ აფხაზური ენის კორუსზე და, ასევე, დეტალურად გავშუქებთ მის უკვე აგებულ საცდელ ვერსიას.

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[3] კ. ფხაკაძე, მ. ჩიქვინიძე, გ. ჩიჩუა, დ. კურცხალია, ი. ბერიაშვილი, შ. მალიძე, ქართული ინტელექტუალური ვებ-კორპუსი: მიზნები, მეთოდები, რეკომენდაციები, გამოიცა საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრის სამეცნიერო-საგანმანათლებლო ჟურნალის ,,ქართული ენა და ლოგიკა‘‘ დამატების სახით, (2017), 4-320.

პროექტის „ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“ მიზნების, ამოცანებისა და მეთოდების ზოგადი მიმოხილვა

კონსტანტინე ფხაკაძე, შალვა მალიძე, მერაბ ჩიქვინიძე, გიორგი ჩიჩუა, დავით კურცხალია

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საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების სასწავლო-სამეცნიერო ცენტრმა გრძელვადიანი პროექტის „ქართული ენის ტექნოლოგიური ანბანი“ [1] კვლევითი მიზნების გათვალისწინებით შეიმუშავა და შოთა რუსთაველის ეროვნული სამეცნიერო ფონდში საგრანტო დაფინანსებაზე წარადგინა დოქტორანტ შალვა მალიძის ორწლიანი პროექტი „ქართული ენით ევროკავშირში ანუ სადოქტორო თემა - ქართული ჭკვიანი კორპუსის ახალი განმავითარებელი ინსტრუმენტებისა და მეთოდების შემუშავება და არსებულთა გაუმჯობესება“.

სადოქტორო პროექტის სამეცნიერო ხელმძღვანელია საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების ცენტრის დირექტორი, პროფესორი კონსტანტინე ფხაკაძე. ამასთან, პროექტში კონსულტანტების სტატუსით ჩართულნი არიან დოქტორები: ალექსანდრე მასხარაშვილი (საფრანგეთი, ნანსის უნივერსიტეტი), ლაშა აბზიანიძე (ჰოლანდია, ტილბურგის უნივერსიტეტი), გიორგი ჩიჩუა (საქართველო, საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების ცენტრი), მერაბ ჩიქვინიძე (საქართველო, საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების ცენტრი). ხაზგასასმელია ისიც, რომ პროექტის სასაფუძვლო შედეგების მომზადებაში მნიშვნელოვანი წვლილი აქვს შეტანილი დავით კურცხალიას (საქართველო, თბილისის სახელმწიფო უნივერსიტეტის მაგისტრანტი, საქართველოს ტექნიკური უნივერსიტეტის ქართული ენის ტექნოლოგიზების ცენტრის მკვლევარი).

ამგვარად, მოხსენებისას ზოგადად მიმოვიხილავთ პროექტის მიზნებს, ამოცანებსა და მოკლედ გავაშუქებთ აგრეთვე მათი გადაწყვეტის ზოგად თეორულ მეთოდებს [2], [3].

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უმთავრესი ქართული გამოწვევა, საპარლამენტო კონფერენციის „ქართული ენა - 21-ე საუკუნის გამოწვევები" შრომები, 2013, 95-102.

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On Gentzen-type Proof Systems for Minimal Quantum Logic

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Quantum logic originated in the 1930s with John von Neumann's book on the mathematical foundations of quantum mechanics [5] and is characterised by the algebraic structure of projection operators over Hilbert spaces, corresponding to *orthomodular lattices*. These are Boolean lattices without the distributive laws but in which *modularity* is stipulated, expressing the condition that, for a lattice $(L, \sqcup, \sqcap)$, $z \sqsubseteq x$ implies $x \sqcap (y \sqcup z) = (x \sqcap y) \sqcup z$, for all $x, y, z \in L$ (recall that, for any $a, b \in L$, $a \sqsubseteq b$ holds iff $a \sqcap b = a$). *Minimal quantum logic*, on the other hand, is a variant of quantum logic being characterised by *ortholattices*, which are orthomodular lattices without the modularity requirement. Accordingly, minimal quantum logic is also referred to as *orthologic*.

In this work, we study axiomatisations of orthologic in terms of Gentzen-type systems. This kind of proof systems are important for analysing proof search in automated deduction and have first been introduced by Gerhard Gentzen in 1935 [3]. We discuss relations of different calculi for orthologic introduced in the literature [4,1,2] and present generalisations to the predicate-logic case.

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About Methods of Stochastic Integral Representation of Wiener Functionals

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In stochastic process theory, the representation of functionals of Wiener process by stochastic integrals, also known as martingale representation theorem, states that a functional that is measurable with respect to the filtration generated by the Wiener process can be written in terms of Ito's stochastic integral with respect to this process. The theorem only asserts the existence of the representation and does not help to find it explicitly. It is possible in many cases to determine the form of the representation using Malliavin calculus if a functional is Malliavin differentiable. We consider nonsmooth (in Malliavin sense) functionals and have developed some methods of obtaining of constructive martingale representation theorems. The first proof of the martingale representation theorem was implicitly provided by Ito (1951) himself. Many years later, Dellacherie (1974) gave a simple new proof of Ito's theorem using Hilbert space techniques. Many other articles were written afterward on this problem and its applications but one of the pioneer work on explicit descriptions of the integrand is certainly the one by Clark (1970). Those of Haussmann (1979), Ocone (1984), Ocone and Karatzas (1991) and Karatzas, Ocone and Li (1991) were also particularly significant. A nice survey article on the problem of martingale representation was written by Davis (2005).

In spite of the fact that this problem is closely related to important issues in applications, for example finding hedging portfolios in finance, much of the work on the subject did not seem to consider explicitness of the representation as the ultimate goal. In many papers using Malliavin calculus or some kind of differential calculus for stochastic processes, the results are quite general but unsatisfactory from the explicitness point of view: the integrands in the stochastic integral representations always involve predictable projections or conditional expectations and some kind of gradients. Shiryaev and Yor (2003) proposed a method based on Ito's formula to find explicit martingale representation for the running maximum of Wiener process. Even though they consider Clark–Ocone formula as a general way to find stochastic integral representations, they raise the question if it is possible to handle it efficiently even in simple cases.

In all cases described above investigated functionals, were stochastically (in Malliavin sense) smooth. It has turned out that the requirement of smoothness of functional can be weakened by the requirement of smoothness only of its conditional mathematical expectation. We (with Prof. O. Glonti, 2014) considered Wiener functionals which are not stochastically differentiable. In particular, we generalized the Clark–Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method of finding of integrand. Now, we have considered functionals which didn't satisfy even these weakened conditions. To such functionals belong, for example, Lebesgue integral (with respect to time variable) from stochastically non smooth square integrable processes.

Variational–Difference Scheme for Kirchhoff Two-Dimensional Nonlinear Dynamical Equation

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In the present work, we consider the classical nonlinear Kirchhoff string equation and study its two-dimensional generalization. Our goal is to find an approximate solution to the initial-boundary value problem for this equation. To do so, we apply a three-layer symmetrical semi-discrete scheme with respect to time variable, in which the gradient value of a nonlinear term is taken at the middle point. This detail is essential, because the inversion of the linear operator is sufficient for computation of approximate solutions for each time step. The variation method is applied for spatial variables. Sine function and differences of the Legendre polynomials were used as coordinate functions. This choice of Legendre polynomials is also important for numerical realization. This way we obtain a system whose structure does not essentially differ from the corresponding difference equations system, allowing us to use the methods developed for solving difference equations system.

Linear variation problem for one-dimensional Kirchhoff equation (for spatial dimension) is considered, error of approximate solution is estimated, and convergence order considering the number of spatial functions is found. General operator equation is considered for symmetric operators. We prove that the matrix corresponding to its variation system is positively defined, when coordinate functions satisfy some natural conditions that will be specified in this work.

Numerical realization program with corresponding interface was created based on the offered algorithm, and numerical computations were carried out for model problems both for one-dimensional and two-dimensional cases. Based on the obtained theoretical results and numerical computations, the practical conclusions about the stability and convergence of the offered method were drawn.

The Invariance Property of Some Type of Derived Unranked Operators

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We use theory extended with contracted operators to study Bourbaki fundamental theory. We have defined unranked invariance and monotonicity notions and proved corresponding theorems. In particular, we have proved following theorem: Assume type of derivable unranked operators is I, II, II^1. If any operators belonging before derivable operators are invariant, then operator is also invariant.

Result 1. Unranked operators and are invariant operators.

Result 2. Unranked operators and are monotonic operators.

Result 3. Unranked operators and are monotonic with respect to last operand.

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Non-Classical Problems of Statics of Linear Thermoelasticity of Microstretch Materials with Microtemperatures

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The paper investigates non-classical boundary value problems of the theory of linear thermoelasticity of microstretch materials with microtemperatures. The representation formulas given in the paper for a general solution of a homogeneous system of differential equations are expressed in terms of four harmonic and four metaharmonic functions. These formulas are very helpful in solving a lot of particular problems for domains of concrete geometry. An applications of such a formula to a III and IV type boundary value problems for the ball is demonstrated. Uniqueness theorems are probed. Explicit solutions are constructed in the form of absolutely and uniformly convergent series.

Convective Clouds Prediction Based on ARL Aerological Diagrams and Radar Observations Data Analysis

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Dangerous meteorological events have significantly increased on the territory of Georgia over the last decades. In accordance with Georgia's complex Orography, the main part of these events has local character and they mainly are associated with the air mass convective movements. Therefore, in the agenda there is an urgent question about timely prediction of possible convection processes in the non-uniform areas of Georgia. For this, it is necessary to study the thermodynamic condition of the atmosphere in the particular areas and therefore determine the degree of instability of the atmosphere. The presented article examines several cases of strong convective meteorological processes processing on the territory of Georgia based as on per the data of the synoptic maps, as well as on the basis of the Sighnaghi meteorological radar data. Aerological diagrams have been built for the each dangerous meteorological event taken part in the region. The diagrams were designed to measure and assess the thermodynamic state of the atmosphere and the rate of uncertainty of the atmosphere based on the particle method. It has been confirmed that the degree of volatility of the atmosphere for the four days was accurate in conjunction with data received from meteorological radar and synoptic mapping. This fact allows us to predict the atmospheric thermodynamic condition in the specific region on the basis of forecasted aerogical data obtained through the model and evaluate the quality of the convective processes in the local area.

A Note on the Rihaczek Transform and its Friends

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The Rihaczek quadratic representation is defined as the product of the signal $\{f(x)$ with its Fourier transform $\widehat{f}(\omega)$. Symmetrically, we obtain the conjugate Rihaczek representation. These representations give rise to play a key role within the theory of time frequency representations. In this work we study the regularity properties of Rihaczek transform and its friends on some function spaces (such as Lorenz spaces, a kind of modulation spaces).

Some key references are given below.

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Parallel Alogirthm For Kirchhoff's One Non-Linear Problem

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Let us consider the initial boundary value problem:

$$\omega_{tt} - \left(\lambda + \frac{2}{\pi}\int_0^\pi \omega_x^2\,dx\right)\omega_{xx} = f(x,t), \quad 0 < x < \pi, \quad 0 < t \le T, \quad \lambda = const > 0, \tag{1}$$

$$\omega(x,0) = \omega^{(0)}(x), \quad \omega_t(x,0) = \omega^{(1)}(x), \quad \omega(0,t) = \omega(\pi,t) = 0. \tag{2}$$

For the homogeneous case the equation (1) was obtained by Kirchhoff [1] who investigated the dynamic state of a string. A great number of works are devoted to this equation both from the standpoint of its solvability and from the standpoint of construction and investigation of numerical algorithms (see, for example, [2]–[5] and the references cited therein).

We represent the parallel algorithm of task (1), (2) that approximates the exact solution and results obtained by parallel computer. Convergence of approximate solution to the exact solution is also proved. This approach was used before for Timoshenko equations [6].

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Stochastic Variational Inequalities and Optimal Stopping

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We give a new characterization of the Snell envelope of a given process as the unique solution of certain stochastic variational inequality (SVI). This approach leads to several a priori estimates for the Snell envelopes and their components [1].

The valuation for American Contingent Claims (ACC) in general financial market model is considered as an application. The robustness of the optimal portfolio/consumption processes with respect to the payoff function is established [1], [2].

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On the Spaces of Generalized Theta-Series with Quadratic Forms of Five Variables

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We form the basis of the space of spherical polynomials of second order $P(2,Q)$ with respect to some diagonal quadratic form $Q(X)$ of five variables and obtained the basis of the space of generalized theta-series $T(2,Q)$ with spherical polynomial P of second order and quadratic form $Q(X)$.

Let

$$Q(X) = b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2$$

be a quadratic form of five variables. For these form we have proved the following

Theorem. $\dim T(2,Q) = 4$ *and the generalized theta-series:*

$$\vartheta(\tau, P_5, Q) = \sum_{n=1}^{\infty} \Big(\sum_{Q(x)=n} \Big(-\frac{b_{11}}{b_{22}}\, x_1^2 + x_2^2 \Big) \Big) z^n,$$

$$\vartheta(\tau, P_9, Q) = \sum_{n=1}^{\infty} \Big(\sum_{Q(x)=n} \Big(-\frac{b_{11}}{b_{33}}\, x_1^2 + x_3^2 \Big) \Big) z^n,$$

$$\vartheta(\tau, P_{12}, Q) = \sum_{n=1}^{\infty} \Big(\sum_{Q(x)=n} \Big(-\frac{b_{11}}{b_{44}}\, x_1^2 + x_4^2 \Big) \Big) z^n,$$

$$\vartheta(\tau, P_{14}, Q) = \sum_{n=1}^{\infty} \Big(\sum_{Q(x)=n} \Big(-\frac{b_{11}}{b_{55}}\, x_1^2 + x_5^2 \Big) \Big) z^n$$

form the basis of the space $T(2,Q)$.

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Computer Modeling of Multi-Party Elections

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From the point of view of developing the models of elections that we proposed earlier, now consider the general nonlinear mathematical model for four party elections with variable coefficients [1], [2]. This model describes quantity dynamics of voters of ruling and three opposition parties between two elections.

There are following objects in the nonlinear mathematical model that describes this process:

1. State and administrative structures, that try to influence supporters of opposition party and gain their support through utilization of state resources (the administrative structures are obviously supporters of the ruling party).
2. The voters who support a ruling party.
3. The voters who support the first opposition party.
4. The voters who support the second opposition party.
5. The voters who support the third opposition party.

Nonlinear mathematical model is described by taking into consideration voters' activity, demographic factor and possible falsification of the elections.

For numerical solution the software MATLAB is used. Numerical experiments are carried out and results are visualized.

The model of elections are quite relevant from the theoretical as well as practical point of view. The interested parties are encouraged to use widely the given results, calculate parameters and choose the future strategy for achieving the desired goal.

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The Generalized Maxwell's Body when the Constitutive Relationship Contains Conformable Fractional Derivatives

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The generalized Maxwell's body is considered when the constitutive relationship contains conformable fractional derivatives [1]. Direct and return ratios are received. Functions of a relaxation and creep are calculated.

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Solution of the First and Second Boundary Value Problems of Statics of the Theory of Elastic Mixture for a Half and a Fourtly Planes

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For homogeneous equation of statics of the linear theory of elastic mixture in the case of a half and a first planes effective are solved the first (when on the boundary domain is given a displacement vector) and the second(when on the boundary domain is given a stress vector) boundary value problems.

By applying analogous formulas due to Kolosov–Muskhelishvii and the method described in [1] the solutions of the problems in a half-plane – are given in quadratures. Further the solution of the first problem is represented by Poisson type formula.

To solve the problems in a fourtly plane we use the method developed in [2] and the solutions of the problems are given in quadratures.

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Potential Method in the Theory of Thermoviscoelasticity of Binary Mixtures

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The present talk concerns with the linear theory of thermoviscoelasticity of binary mixture which is modelled as a mixture of an isotropic elastic solid and a Kelvin-Voigt material. The basic boundary value problems (BVPs) of quasi statics of the considered theory are investigated and some basic results of the classical theory of thermoviscoelasticity are generalized. The fundamental solution of the system of equations of quasi-statics is constructed by elementary functions. The representation of Galerkin type solution is obtained. The Green's formulae and the formulae of integral representation of regular vector and regular (classical) solutions are obtained. The uniqueness theorems for solutions of the internal and external basic BVPs are proved. The basic properties of potentials and singular integral operators are presented. Finally, the existence theorems for the above mentioned BVPs are proved by means of the potential method and the theory of singular integral equations.

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On the Solution of the Neumann BVP of Thermo-Electro-Magneto Elasticity for Half Space

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Let $\mathbb{R}^3$ be divided by some plane into two half-spaces. Assume that these half-spaces are $\mathbb{R}^3_1 := \{x \mid x=(x_1,x_2,x_3)\in\mathbb{R}^3,\ x_3>0\}$, $\mathbb{R}^3_2 := \{x \mid x=(x_1,x_2,x_3)\in\mathbb{R}^3,\ x_3<0\}$.

We investigate the following Neumann boundary value problem of the thermo-electro-magneto-elasticity theory for a half-space.

Neumann problem. Find a solution vector $U=(u,\varphi,\psi,\vartheta)^\top\in[C^1(\overline{\mathbb{R}^3_{1,2}})]^6\cap[C^2(\mathbb{R}^3_{1,2})]^6$ to the system of equations

$$A(\partial)U=0 \ \text{ in } \ \mathbb{R}^3_{1,2} \tag{1}$$

satisfying the Neumann type boundary condition

$$\{\mathcal{T}(\partial,n)U\}^{\pm}=F \ \text{ on } \ S=\partial\mathbb{R}^3_{1,2}, \tag{2}$$

where $A(\partial)=[A_{pq}(\partial)]_{6\times 6}$ is the matrix differential operator of statics in the theory of thermo-electro-magneto-elasticity and $\mathcal{T}(\partial,n)$ is the corresponding generalised stress operator [1]. We require that $F\in\overset{\circ}{C}{}^\infty(\mathbb{R}^2)$.

Theorem 1. *The Neumann boundary value problems* (1), (2) *have at most one solution* $U=(u,\varphi,\psi,\theta)^\top$ *in the space* $[C^1(\overline{\mathbb{R}^3_{1,2}})]^6\cap[C^2(\mathbb{R}^3_{1,2})]^6$ *provided*

$$\theta(x)=\mathcal{O}(|x|^{-1}) \quad \text{and} \quad \partial^\alpha\widetilde{U}(x)=\mathcal{O}(|x|^{-1-|\alpha|}\ln|x|) \ \text{ as } \ |x|\to\infty$$

for arbitrary multi-index $\alpha=(\alpha_1,\alpha_2,\alpha_3)$. *Here* $\widetilde{U}=(u,\varphi,\psi)^\top$.

Theorem 2. *Let* $F\in\overset{\circ}{C}{}^\infty(\mathbb{R}^2)$ *and* $\int\limits_{\mathbb{R}^2} F(\widetilde{x})\widetilde{x}^\beta\,d\widetilde{x}=0$ *for arbitrary multi-index* $\beta=(\beta_1,\beta_2)$, $|\beta|=0,1,2$. *Then the unique solutions of the boundary value problems* (1), (2) *can be represented in the form*

$$U(x)=\mathcal{F}^{-1}_{\widetilde{\xi}\to\widetilde{x}}\Big[\mathcal{T}(-i\xi,n)\Phi^{(-)}(\widetilde{\xi},x_3)[\Phi^{(-)}(\widetilde{\xi},0)]^{-1}\widehat{f}(\widetilde{\xi})\Big], \ \ x_3>0, \quad \text{or}$$

$$U(x)=\mathcal{F}^{-1}_{\widetilde{\xi}\to\widetilde{x}}\Big[\mathcal{T}(-i\xi,n)\Phi^{(+)}(\widetilde{\xi},x_3)[\Phi^{(+)}(\widetilde{\xi},0)]^{-1}\widehat{f}(\widetilde{\xi})\Big], \ \ x_3<0.$$

Here $\mathcal{F}^{-1}$ denotes the inverse generalized Fourier transform and $\Phi^{(\pm)}$ are matrices constructed by the symbol matrix of the operator $A(\partial)$ [2].

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On Coverings and Decompositions of Subsets of Euclidean Space

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In this presentation we consider various coverings of the Euclidean plane and Euclidean space which are produced by using a set-theoretical approach. Special attention is paid to so-called homogenous coverings which are constructed with the aid of Axiom of Choice and method of transfinite induction. In this context, it should be noted that in 1985 A. B. Kharazishvili solved a well-known problem concerning homogenous coverings of the Euclidean plane and three-dimensional Euclidean space with congruent circumferences (see [1]). We apply the method similar to that of [1]. In our joint paper (see [2]), some questions related to indicated problems are highlighted too. We also consider several decompositions of certain subsets of Euclidean space and demonstrate their close connections with independent families of sets. Some applications of special decompositions in mathematical analysis are also discussed.

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Features of the Analytic Functions' Border Meanings in the Unit Disk

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Generally, there is no connection between the angular border meanings of the analytic functions' sequences and the angular meanings of the sequences' border functions. Even if the members of the sequence are multinomials. However, there are necessary and sufficient conditions, such that, on the positively measured subsequence of the unit circle there is a sequence, which is uniformly convergent to the analytic function inside unit circle and is convergent to the given function almost everywhere on the borders of the circle. Besides, the angular border meanings of the border functions are almost equal to the meanings of the function.

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About Backward Stochastic Integral

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We study the so called backward stochastic integral which in one specific case has been entered by McKean (1969), but we consider a case when the integrand in addition depends on random and time variables and some properties of this stochastic integral construction are studied. As it is known, in ordinary integration theory the requirement of measurability of integrand is essentially less restriction, than the condition of its integrability, which demands a certain bound condition of absolute value of the integrand.

As for the Ito's Stochastic Integral $\int\limits_0^T f(t,\omega)\,dW_t(\omega)$, here, the situation is opposite in some sense: moreover that, integrand $f(t,\omega)$ is a measurable function of two variables, it should be an adapted process, i.e. for all $t\in[0,T]$ the random variable $f(t,\cdot)$ must be measurable with respect to the σ-algebra $\Im_t^W := \sigma\{W_s:\ s\in[0,t]\}$, which means that integrand should be independent of the future increments of the Wiener process. On the one hand, it is clear that in many cases this is natural condition, where the filtration $\Im_t^W$ represents the evolution of the available information. On the other hand, this condition for a long time was limiting, both development of the theory, as well as applications of stochastic calculus.

In the 70th of the last century, there have been a lot of tries to weaken the adaptedness requirement for the integrand of Ito's stochastic integral, as well as in the theory of filtration expansion. A completely different approach has been suggested by Skorokhod (1975), which didn't demand independence of the integrand from the future increments of the Wiener integrator and was symmetrical with respect to time reversal. For this purpose it was required to demand smoothness of an integrand in some sense, in particular its stochastically differentiability.

We define and study some properties of the backward stochastic integral where the integrand (in difference from the case considered in [1]) depends on random and time variables. In particular, the class of integrable in such sense integrands is described, the relationships with both of Ito's and Stratanovich's integrals are established and the change of variables rule (the analogous of Ito's formula) is given. As it is known, if for every period of time f is dependent of Wiener process past behavior (or in other words, it is adapted process to its own natural filtration), then the integral $\int\limits_0^T f\,dW$ had been defined by Ito and it has some good properties, but if f is dependent of Wiener's process future behavior, then many properties (e.g., isometry and martingale property) are lost.

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About the Fast Direct Solution of DGTD Equation

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In recent years the discontinuous Galerkin time domain method (DGTD) [1] is widely used for applications arising in electromagnetics. In principle, DGTD is a variant of conventional finite element method (FEM) [2]. The main difference is that in DGTD the basis functions are only defined on a single element without any overlap with the neighboring elements. This effectively decouples the elements. After having performed all expensive operations on each element separately, the coupling between the elements is reintroduced via the so-called numerical flux. Usage of the spatial discretization based on unstructured, tetrahedral finite elements is one of the main advantages of DGTD. The explicit time integration schemes, such as Runge-Kutta and Leap-Frog are easy applicable, but unfortunately computationally expensive due to restriction of time step size (Courant–Friedrichs–Lewy condition). Thus it is required to use implicit time stepping scheme that is unconditionally stable and allows usage of larger time step, which in turn reduces the calculation time. However, efficiency of this approach depends on inversion of huge sparse matrices. This report presents an optimized implicit time integration scheme for DGTD based on fast direct inversion of matrix.

$$Au^{n+1} = Bu^n + Cv^{inc},$$

where A, B and C are sparse matrices, u^n is vector of fields calculated at previous time step, v^{inc} is vector of incident fields and u^{n+1} is unknown vector that should be calculated.

For solution of the equation the same approach is used as in Fast Direct Solver (FDS), which was reported on the last conference [3]. Along with full and compressed blocks we introduce zero blocks, and newly obtained during solution non-zero blocks we compress by SVD method

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MHD-Flow of Conducting Liquid in Ducts with Arbitrary Conductivity of Walls

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The paper deals with steady flow of conducting liquid in rectangular ducts with arbitrary conductivity of walls in the presence of transverse magnetic field the variational method of investigation being used. The obtained results are applied particularly for the determination of viscous and Joule losses in the MHD-converter.

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On the Infinite Sequence of Twin Prime Numbers

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Consider twin prime numbers 17 and 19. We construct two infinite sequence of numbers

107, 1007, 10007, 100007 and so on;

109, 1009, 10009, 100009 and so on.

By using mathematical induction is proved that all members of the first sequence are prime. Members a_4, a_{10}, a_{16}, a_{22} and so on of the second sequence are composite and other members are prime. From these follows that there exists the infinite sequence of twin prime numbers.

I-Rad-⊕-Supplemented Modules

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Let M be an R-module and let I be an ideal of R. We say that M is I-Rad-⊕-*supplemented modules*, provided for every submodule N of M, there exists a direct summand K of M such that $M = N + K$, $N \cap K \subseteq IK$ and $N \cap K \subseteq \mathrm{Rad}(K)$. The aim of this paper is to show new properties of I-Rad-⊕-supplemented modules. Especially, we show that any finite direct sum of I-Rad-⊕-supplemented modules is I-Rad-⊕-supplemented. We also prove that an R-module M is I-Rad-⊕-supplemented if and only if K and $\frac{M}{K}$ are I-Rad-⊕-supplemented for a fully invariant direct summand K of M. Finally, we obtain the following result: Let R be a Dedekind domain which is not a field and M be an injective R-module. Then M is Rad-⊕-supplemented if and only if M is I-Rad-⊕-supplemented for every non-zero ideal I of R.

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Absolutely Convergence Factors for $Lip\,1$ Class Fourier Coefficients

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It is known that if (φ_n) is either trigonometric, Haar or Walsh systems and $f(x) \in Lip\,\alpha$, $\alpha \in (0,1]$, then

$$\sum_{n=1}^{\infty} |c_n(f)| n^{\gamma} < +\infty,$$

where $\gamma < \frac{\alpha}{2}$ and $c_n(f) = \int\limits_0^1 f(x)\varphi_n(x)\,dx$.

This talk is devoted to investigate sequences, for which multiplication with $\sum\limits_{n=1}^{\infty} |c_n(f)|n^{\frac{1}{2}}$ provides absolute convergence of $Lip\,1$ class Fourier coefficients.

We present theorem, which is a criterion for which above mentioned numerical sequences are absolute convergence factors of $Lip\,1$ class Fourier coefficients.

Hardy Operators in Grand Lebesgue Spaces

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Grand Lebesgue spaces over sets of infinite measure are defined with using an additional characteristic $a(\cdot)$ called a grandizer. Conditions on the grandizer $a(x)$ for the Hardy operators to be bounded in the grand Lebesgue spaces $L^{p)}_a(\mathbb{R}^n)$ are found, and the lower and upper estimates for a sharp constant in the one-dimensional and multidimensional Hardy inequalities are given in dependence on the grandizer. For some special choice of the grandizer it is proved that this sharp constant is equal to the sharp constant for the classical Lebesgue spaces.

The Number of Representations of Some Positive Integers by Binary Forms

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It is well known how to obtain the formulas for the number of representations of a positive integer by a binary quadratic form which belongs to one-class genera. In this paper we give a full description of binary forms belonging to multi-class genera for which the problem of obtaining formulas for the number of representations of some positive integers can be easily reduced to the case of one-class genera.

On Cross-Sections in the $(2,0)$-Semitensor Bundle

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Using projection (submersion) of the tangent bundle TM over a manifold M, we de ne a semi-tensor (pull-back) bundle tM of type (2,0). The main purpose of this paper is to investigate complete and horizontal lift of vector fields for semi-tensor (pull-back) bundle tM of type (2,0). In this context cross-sections in a special class of semi-tensor (pull-back) bundle tM of type (2,0) can be also defined.

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An Application of Haar–Wavelet Collocation Method to a Beam Equation

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In this study, An Application of Haar-Wavelet Collocation Method to a Beam Equation is presented. In order to show the efficiency and robustness of the method, some numerical examples are given by means of figures.

Solitary Wave Solutions of a Special Class of Calogero–Degasperis–Fokas Equation

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In this study, G'/G expansion method is applied for a new solitary wave solutions of a special class of Calogero–Degasperis–Fokas equation. New periodic and solitary wave solutions for the nonlinear equation are derived.

On the Grammatical, Set Interpretations and Unification of Denotations of Logical Operations

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In the papers [1]–[5] there were discussed classification of logical operations receiving from using "or", "and", and "just or" conjunctions on three P, Q and R sentences paired, alternatively and multiple ways. As well their grammatical, set interpretations and circuits in Digital Electronics. This paper discusses similar circuits in case of conditional sentences operation (9 proposition), circuits of negations of the logical operations (21 circuits) and their grammatical, set interpretations. As well equivalent representations will be also given. This kind of grammatical interpretations is not discussed in English, in German and in Russian languages. Well-known denotations of classic operations will be given and possibility of changing them will be discussed.

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The Consistent Estimators for Statistical Structures

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Let there is given (E, S) measurable space and on this space there given $\{\mu_i, i \in I\}$ family of probability measures defined on S, the I is set of parameters.

Definition 1. A statistical structure is called the set of objects $\{E, S, \mu_i,\ i \in I\}$, where (E, S) is a measurable space and $\{\mu_i,\ i \in I\}$ is a family of probability measures on it.

Definition 2. A statistical structure $\{E, S, \mu_i,\ i \in I\}$ is called strongly separable if there exists disjoint family of S-measurable sets $\{X_i,\ i \in I\}$ such that the relations are fulfilled:

$$\mu_i(X_i) = 1 \ \ \forall\, i \in I.$$

Definition 3. A linear subset $M_B \subset M^\sigma$ is called a Banach space of measures if:

1) one can define a norm on M_B under which M_B will be a Banach space and for any orthogonal measures μ, $\nu \in M_B$, and real number $\lambda \neq 0$ the equality $\|\mu+\lambda\nu\| \geq \|\mu\|$ is fulfilled;
2) if $\mu \in M_B$, $|f(x)| \leq 1$, then $\nu_f(A) = \int\limits_A f(x)\mu\,(dx) \in M_B$ and $\|\nu_f\| \leq \|\mu\|$;
3) if $\nu_n \in M_B$, $\nu_n > 0$, $\nu_n(E) < +\infty$ $(\forall\, n \in N)$ and $\nu_n \downarrow 0$ then for any linear functional $l^* \in M_B^* : \lim\limits_{n\to\infty} l^*(\nu_n) = 0$.

Let $\xi_i(t)$ $(i \in I,\ t \in T \subset R^n)$ be the Gaussian real homogeneous field on T with zero mean and correlation function $R_i(t-s)$, $i \in I$; $t, s \in T$. Let

$$\int\limits_{R^n}\int\limits_{R^n} \frac{|\widetilde{b}_{i,j}(\lambda,\mu)|^2}{f_i(\lambda)f_j(\mu)}\,d\lambda\,d\mu = +\infty \ \ \forall\, i,j \in I,$$

where $\widetilde{b}_{i,j}(\lambda,\mu)$ is Fourier transformation of $b_{i,j}(t,s) = R_i(t,s) - R_j(,st)$, $i,j \in I$. Then the corresponding probability measures μ_i and μ_j are pairwise orthogonal.

Definition 4. Let's say that the statistical structure $\{E, S, \mu_i,\ i \in I\}$ admits a consistent estimators of parameters $i \in I$, if there exists at least one measurable map $\delta : (E,S) \to (I, B(I))$, such that $\overline{\mu}_i(\{x :\ \delta(x)\}) = 1\ \forall\, i \in I$.

Theorem. *Let $M_B = \bigoplus\limits_{i\in I} M_{B_i}$ be a Banach space of measures; E be the complete metric space whose topological weights are not measurable in a wider sense; S be the Borel σ-algebra in E and $cardI \leq c$. Then in the theory* (ZFC) & (MA) *the Gaussian orthogonal statistical structure $\{E, S, \mu_i,\ i \in I\}$ admits a consistents estimators of $i \in I$ if and only if the correspondence $f \to l_f$ defined by the equality $\int\limits_E f(x)\mu_j\,(dx) = l_f(\mu_j)$, $\mu_j \in M_B$, is one-to-one* (*here l_f is linear functional on M_B f*).

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Numerical Simulation of Some Non-Classical Elasticity Problems for half-Plane by Boundary Element Method

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The paper set a non-classical problems, which formulate in the following way: what normal stress supposed to be applied on part of the boundary, that at the segment lying inside the homogeneous isotropic elastic half-plane to obtain a pre-given conditions. The problems solved by Boundary element method [1]. Testable examples are given, which shows us, that what the normal stress supposed to be applied on part of the boundary, that at the segment lying inside the body to obtain a pre-given stress or displacement. Represented the numerical results, appropriate graphics, mechanical and physical interpretation this problems.

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About Mathematics Teacher to Assess the Work Performed by the Student

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Today, when it is important to move on to the new system of assessment of the teacher's activity, it is especially noteworthy how the teacher evaluates the task performed by the pupil. An assessment of pupil by the teacher is a necessary component of learning and teaching quality management. Correct assessment is equally important for both parties involved in the teaching process. A last kind of task from the teacher's subject examination is presented in the work. In particular, assessment of the work performed by the pupil. During the assessment the teacher finds faults and gives pupil relevant recommendations.

Definition of Deflected Mode of Anisotropical Cylindrical Body at the Irregular Temperature Influence

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As it is known, determination of tensed-deformated condition of log of cylindrical shape belongs to the group of spatial objectives of mechanics of deformative bodies. The spatial objectives like this, on the basis of the classical elasticity, as a rule is accompanied to solve the flat objectives with the help of several hypothetic assumptions(among them the principle of Saint-Venant), with what, it is evident, their exact solving is not reached. In the work the task of deflected anisotropic cylindrical circular-section body, mounted on either end motionlessly is studied at the irregular temperature influence. Without the usage of hypothetical assumptions (among them Saint-Venant principle) the exact solution of the task is given with the collocation method. The coordinate system without dimension is used in cylindrical coordinates. With the help of corresponding transformation of set objectives, basic equations of theory of elasticity are portrayed. The internal strain and the relocate components are found, which satisfy the corresponding boundary conditions of the task, also the equilibrium equations and physical equations.

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